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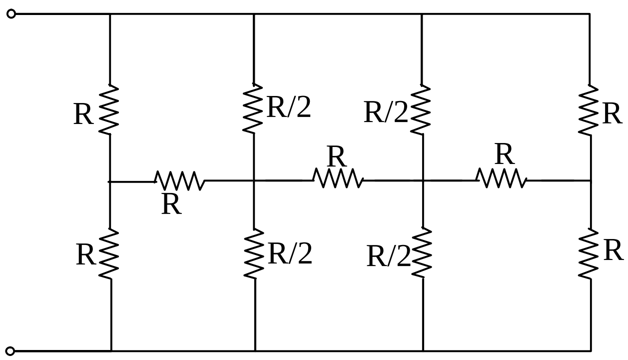
GATE – 2015 – Electronics & Communication Engineering (EC)

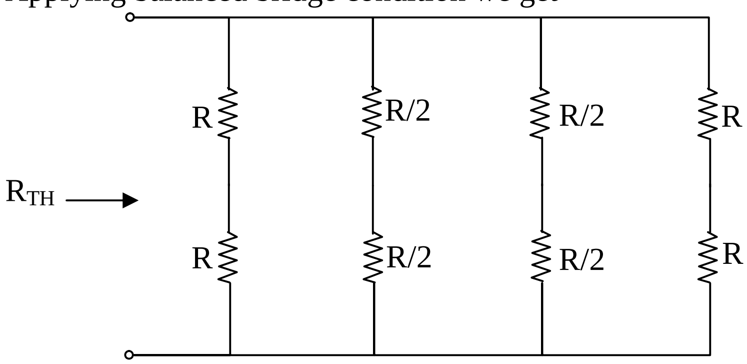
SOLUTIONS

SET – 1 (31st January Forenoon Session)

01.Ans: 100
Sol:



Applying balanced bridge condition we get



$R_{TH} = \frac{R}{3} = \frac{300}{3}$ $\Rightarrow R_{TH} = 100\ \Omega$

02.Ans: (25)
Sol: At resonant frequency the magnitude of voltage across capacitor is

$\Rightarrow V_c = QV\angle{-90^o}$

$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1\times10^{-3}\times1\times10^{-6}}} = 1\times10^5$ rad/s; $Q = \frac{\omega_o L}{R}$

$V_c = 2.5\times10\angle{-90^o} = 25V$

03.Ans: (b)

Sol: The characteristic equation for series RLC circuit is

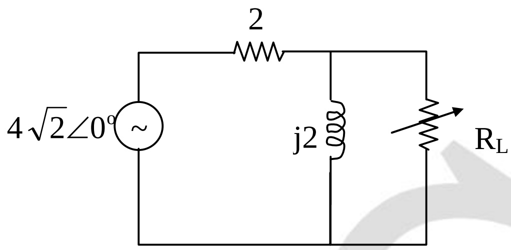
$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \text{ -------- (1)}$$

Standard characteristic equation for any 2nd order system is $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ ------- (2)

Comparing (1) & (2) we get $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

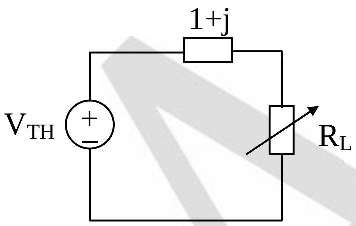
04.Ans: 1.656

Sol: $V_m = \sqrt{2}\,V_{rms} = 4\sqrt{2}$ V



$Z_{TH} = j2 \,//\, 2 = \frac{2\times j2}{2+j2} = 1 + j$

$V_{TH} = \frac{j2\times 4\sqrt{2}}{2+j2} = 4\angle 45^o$



$R_L = \sqrt{1^2 + 1^2}$

$R_L = \sqrt{2}\ \Omega$

$$V_{R_L} = \frac{R_L V_{TH}}{1 + j + R_L} = \frac{(4\angle 45^o)(\sqrt{2})}{1 + \sqrt{2} + j}$$

$V_{R_L} = 2.1647\ \angle 22.5^o$

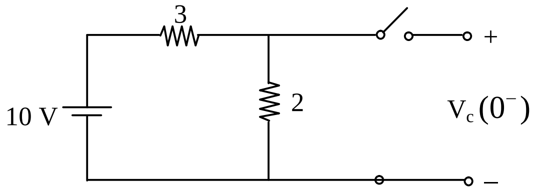
$I_{R_L} = 1.53\angle 22.5$

$P_{R_L} = V_{rms}\, I_{rms} \cos(\theta - \phi)$

$$= \frac{2.1647\times 1.53}{2}\cos 0 = 1.6567\text{W}$$

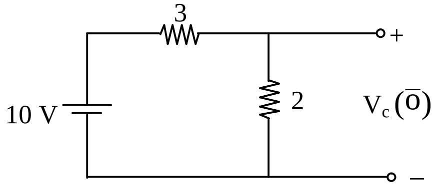
05. **Ans: 2.528V**

Sol: The circuit at t = 0 is



$V_C(0) = V_C(0^-) = V_C(0^+) = 0V$

The circuit at t = ∞



$$V_C(\infty) = \frac{2}{2+3} \times 10 = 4V$$

τ = RC = 3/2 (5/6)

τ = 1 sec

$V_C(t) = V_C(\infty) + [V_C(0) - V_C(\infty)]\, e^{-t/c}$

$\therefore V_C(t) = 4 + [0 - 4]e^{-t}$

$V_C(t) = 4(1 - e^{-t})$ V

At t = 1 sec

$\therefore V_C(1) = 4(1 - e^{-1}) = 2.528$ V

$V_C(1) = 2.528$ V.

06. **Ans: (c)**

Sol: ∵ $V_C(\infty) = 3$ V (i.e) capacitor

Would have reached final steady state value, in this process 50% of energy is also dissipated in the resistor as heat while changing capacitor.

$E_{absorbed} = E_{stored\ (capacitor)} + E_{dissipated\ (resistor)}$

$V_c(t) = V_s[1 - e^{-t/\tau}]$ ……… (1)

$$i_c(t) = \frac{CdV_c}{dt} = CV_s\left[\frac{1}{RC}\right]e^{-t/\tau}$$

$$i_c(t) = \frac{V_s}{R}e^{-t/\tau} \text{ ………. (2)}$$

$P_c(t) = V_c(t) \cdot i_c(t)$

$$P_c(t) = \frac{V_s^2}{R}[e^{-t/\tau} - e^{-2t/\tau}] \text{ ……. (3)}$$

$$E_c(t) = \int_0^\infty p_c(t)\,dt$$

Energy stored upto t = ∞

is energy at t = ∞

$= \frac{1}{2}[v(\infty)]^2$

$= \frac{1}{2} C\ V_s^2 = 0.45$ J; $i_R(t) = i_c(t)$

$i_R(t) = \frac{V_s}{R}\ e^{-t/\tau}$

$V_R(t) = V_s\ e^{-t/\tau}$

$P_R(t) = \frac{V_s^2}{R}\ e^{-2t/\tau}$

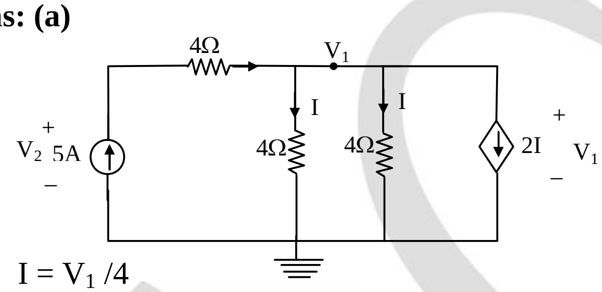
$E_R(t) = \int_0^{\infty} P_R(t)dt$

$= \frac{1}{2} C\ V_s^2 = 0.45$ J

So, total energy taken from source = 0.9 J

07.**Ans: (a)**

Sol:



$I = V_1/4$

Applying KCL at node A we have

$-5 + I + I + 2I = 0$

$4I = 5$

$I = \frac{5}{4}$ A

$V_1 = 4I = 5V$

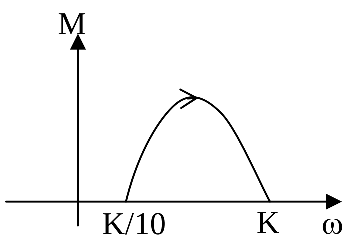
$V_2 - 4\times 5 = 5V \Rightarrow V_2 = 25$ V

08. Ans: (a) I^st Quadrant

Sol: $M = \frac{K\sqrt{\omega^2+1}}{\sqrt{\omega^2+100}}$

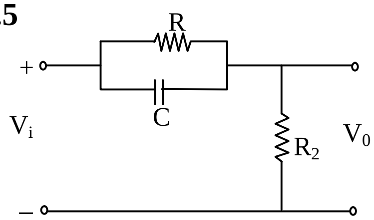
$\phi = \tan^{-1}\omega - \tan^{-1}\frac{\omega}{10}$

ω	M	ϕ
0	K/10	0
10	0.7106K	39.29
.	.	.
∞	K	0



09.Ans: 0.5

Sol:



Standard TF of a lead n/w is

$$G(S) = \alpha \frac{(1+s\tau)}{(1+\alpha s\tau)} \text{ —— (1)}$$

Given TF function is $\frac{s+2}{s+4}$ —— (2)

Comparing (1) & (2)

$\tau = 0.5$; $\alpha\tau = 0.25$

$\Rightarrow \alpha = 0.5$

$\therefore RC = \tau = 0.5$

10. Ans : (a)

Sol : $G(s)H(s) = \left(K_P + \frac{K_I}{s}\right)\left(\frac{1}{s(s+5)}\right)$

$$G(s)H(s) = \frac{(sK_P + K_I)}{s^2(s+5)}$$

CE $\Rightarrow 1+G(s)H(s) = 0$

$s^3+5s^2+sK_P+K_I = 0$

s^3	1	K_P
s^2	5	K_I
s^1	$\frac{5K_P - K_I}{5}$	
s^0	K_I	

for stability $K_I > 0$ $5K_P > K_I$

$\therefore 5K_P > K_I > 0$

11. Ans : (b)

Sol : Negative feedback in a closed loop control system reduces the overall gain of the system. As the gain-bandwidth product of any system should be constant, so the bandwidth should increase. Negative feedback improves distance rejection and it reduces the sensitivity to parameter variation.

12. Ans: K = 25.54

Sol: $G(s)H(s) = \frac{K(s+4)}{(s+8)(s^2-9)}$

Any point lies on the root locus if it satisfies the angle criteria. The point satisfying angle criteria definitely satisfies magnitude criteria.

Angle criteria $\angle G(s)H(s) = (2q+1)180^o$

Magnitude criteria $|G(s)H(s)| = 1$

$\angle G(s)H(s) = \{\angle(-1+j2+4) - [\angle(-1+j2+8) + \angle(-1+2j)^2 - 9]\}$

$\approx 180^o$

$\therefore$ the point satisfies angle criteria

$$\therefore \left|\frac{K(-1+j2+4)}{(-1+j2+8)[(-1+j2)^2-9]}\right| = 1$$

$$\left|\frac{K\left(\sqrt{13}\right)}{\sqrt{53}\sqrt{60}}\right| = 1$$

$\Rightarrow K = 25.54$

13. Ans: 12

Sol: Characteristic equation is

$S^3 + 4S^2 + 3S + K = 0$

By Routh criteria condition for marginal stability is k = 12.

14. Ans: 0.52

Sol: $\mu_n = 1200\,cm^2/Vs$;

$\mu_p = 400\,cm^2/Vs$;

$N_D = 1\times10^{16}/cm^3$

$\sigma = n_n q\mu_n + p_n q\mu_p$

For Si $n_i = 1.5\times10^{10}$; $n_n = N_D = 1\times10^{16}/cm^3$

$$p_n = \frac{n_i^2}{N_D} = \frac{(1.5\times10^{10})^2}{1\times10^{16}} = 22500$$

As $n_n >> p_n$

$\therefore\ \sigma = N_D q\mu_n = (1\times10^{16})(1.6\times10^{-19})(1200)$

$= 1.92\ (\Omega\text{-cm})^{-1}$

$\rho = 0.52\Omega\text{-cm}$

15. Ans: (a)

Sol: Negative differential resistance is exhibited in P-N junction diode which are heavily doped on both sides

Eg: Tunnel diode.

16. Ans: 2.5

Sol: $C_J = \dfrac{C_{Jo}}{\left(1+\dfrac{V_R}{V_o}\right)^m}$; $m = \dfrac{1}{2}$ for abrupt junction.

$$\frac{C_{J1}}{C_{J2}} = \frac{\left(1+\frac{V_{R2}}{V_o}\right)^{1/2}}{\left(1+\frac{V_{R1}}{V_o}\right)^{m}} = \frac{\left(1+\frac{7.25}{0.75}\right)^{1/2}}{\left(1+\frac{1.25}{0.75}\right)^{1/2}}$$

$$\frac{C_{J1}}{C_{J2}} = 2 \Rightarrow C_{J2} = \frac{5}{2} = 2.5$$

$$\Rightarrow C_{J2} = 2.5\text{pF}$$

18.Ans:(a)

Sol: $V_D = V_G = V_{DD}$

$V_{DS} = V_{GS} = V_{DD} - V_{SS}$

$I_D = K[V_{GS} - V_T]^2$

$I_D = K[V_G - V_{SS} - V_T]^2$

$I_D = K_1[(V_{DD} - V_T) - V_{SS}]^2$

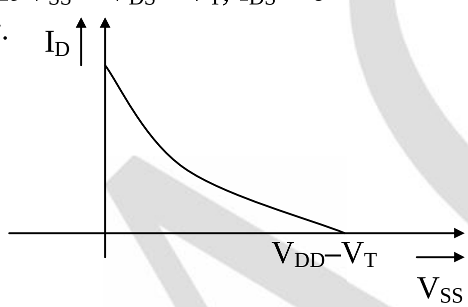
$$\frac{\partial I_D}{\partial V_{GSS}} = 2K_1[V_{DD} - V_T - V_{SS}]$$

$$= 2K_1[K_2 - V_{SS}]$$

$\Rightarrow$ Slope of the curve is negative

At $V_{SS} = V_{DS} - V_T$; $I_{DS} = 0$

$\therefore$



19. Ans: 20.50 kt

Sol: Given $\lambda = 0.05V^{-1}$

$I_D = 1\text{mA}$; $V_{DS} = 0.5$ V

$I_D = k[V_{GS}-V_T]^2 (1+\lambda V_{DS})$ ------- (1)

$$k[V_{GS}-V_T]^2 = \frac{I_D}{1+\lambda V_{DS}}$$

$$= \frac{1\times 10^{-3}}{1+0.05\times 0.5}$$

$$k[V_{GS}-V_T]^2 = \frac{1}{1025} \quad \text{--------- (2)}$$

output resistance $r_o = \left.\frac{\partial V_{DS}}{\partial I_D}\right|_{V_{GS}\to \text{const}}$

Differentiating (1) we get

$$\frac{\partial I_D}{\partial V_{DS}} = k[V_{GS}-V_T]^2 \lambda \quad \text{------- (3)}$$

Substituting (2) in (3) we get

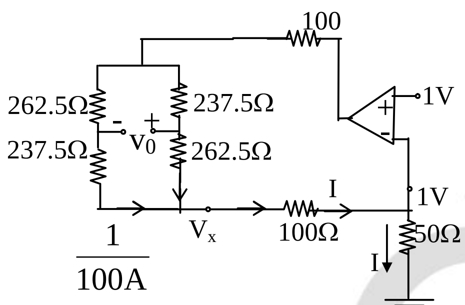
$$\frac{\partial I_D}{\partial V_{DS}} = \frac{1}{1025} \times 0.05$$

$$\frac{\partial V_{DS}}{\partial I_D} = r_o = \frac{1}{4.878 \times 10^{-5}}$$

$r_o = 20.50\ k\Omega$

20. Ans: 250

Sol:



By virtual ground concept $V_i^+ = V_i^- = 1V$

$I = \frac{1}{50} A$

$V_x = 100\ I + 1$

$= \frac{100}{50} + 1$

$= 3\ V$

$V_o^+ - 262.5 \times \frac{1}{100} = 3$

$V_o^+ = 5.625\ V$

$V_o^- = \frac{237.5}{100} + 3V = 5.375\ V$

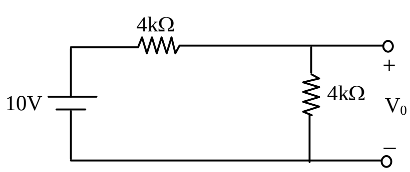
$V_o = V_o^+ - V_o^-$

$V_o = 0.25\ V$

$V_o = 250\ mV$

21. Ans: 5

Sol: The region of operation of the zener diode should be determined first. Removing the zener diode from the circuit above we have the circuit as



$V_0 = \frac{4}{4+4} \times 10 = 5V$ as the voltage across the zener is less than 6 V so the zener diode does not operate in reverse break down region.

$\therefore\ V_0 = 5$ V

22. Ans : 83.85

Sol : $I_D = I_S e^{\frac{V_{BE}}{V_t}}$

$$V_{BE} = V_t \ln\left[\frac{I_D}{I_S}\right]$$

From the given figure

$$V_{12} = V_1 - V_2$$

$$= V_t \ln\left[\frac{I_{D1}}{I_S}\right] - V_t \ln\left[\frac{I_{D2}}{I_S}\right]$$

$$= V_t\left[\ln\left(\frac{I_{D1}}{I_{D2}}\right)\right] \text{---------------(1)}$$

It is known that

$$V_t = \frac{T}{11600}$$

$$\frac{V_{t_1}}{T_1} = \frac{V_{t_2}}{T_2}$$

$$V_{t_2} = \frac{T_2}{T_1} \times V_{t_1}$$

$$= \frac{323}{300}(26\,m)$$

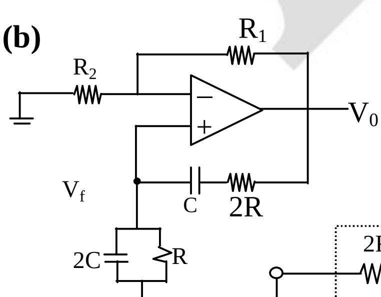
$V_{t_2} = 27.99\,mV$ -------------(2)

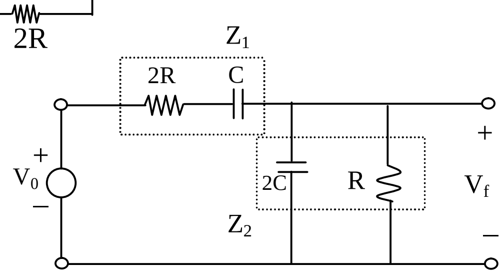
Substitute the above value in equation (1)

$$\therefore\ V_{12} = 27.99\ m \times \text{In}\left(\frac{80m}{4m}\right) \Rightarrow V_{12} = 83.85\ mV.$$

23. Ans : (b)

Sol :





$$Z_1 = 2R + \frac{1}{SC}$$

$$= \frac{1+2SRC}{SC}$$

$$Z_2 = \frac{R \times \frac{1}{2SC}}{R + \frac{1}{2SC}} = \frac{R}{2SRC+1}$$

$$= \frac{R}{R + \frac{1}{2SC}}$$

$$\therefore \quad V_f = V_0 \times \frac{Z_2}{Z_1 + Z_2}$$

$$= V_0 \times \frac{\frac{R}{1+2SRC}}{\frac{1+2SRC}{SC} + \frac{R}{1+2SRC}}$$

$$= V_0 \frac{R}{(1+2SRC)} \times \frac{SC(1+2SRC)}{(1+2SRC)^2 + SRC}$$

$$= \frac{V_0 RCS}{1+4SRC+4S^2R^2C^2+SRC}$$

$$V_f = \frac{V_0 RCS}{1+5RCS+4R^2C^2S^2}$$

$$\beta = \frac{V_f}{V_0} = \frac{RSC}{1+5RCS+4R^2C^2S^2}$$

$$\text{Gain } A = 1 + \frac{R_1}{R_2}$$

For oscillations to occur

$A\beta = 1$

$$\Rightarrow \quad A = \frac{1}{\beta}$$

$$1 + \frac{R_1}{R_2} = \frac{1+5RCs+4R^2C^2s^2}{sRC}$$

Equating Real & Imaginary parts

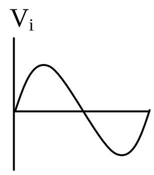
$1+\frac{R_1}{R_2}=5$ | $\frac{1}{RCs}+4RCs=0$

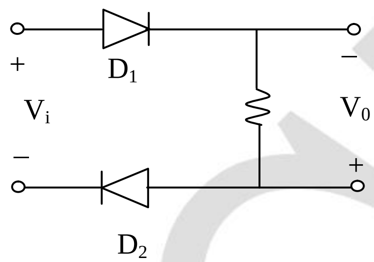
$\frac{R_1}{R_2}=4$ | $4R^2s^2C^2+1=0$

$R_1 = 4R_2$ | $\omega_o = \frac{1}{2RC}$

24. Ans : (c)

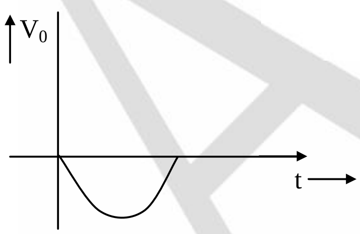
Sol: The given circuit diagram can be redrawn as





(a) For +ve half cycle :
For +ve half cycle the diodes D_1 and D_2 are ON
$V_0 = -V_{in}$

(b) For –ve half cycle :
For –ve half cycle the diodes D_1 and D_2 are off



25. Ans: (b)

Sol: The result of addition operation is stored in accumulator (A) and the status of carry bit is stored in the PSW i.e flag register (F).

26. Ans: (a)

Sol: $F = xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}y\bar{z}$

	00	01	11	10
0	0	0	0	1
1	1	1	0	1

$F = (x+y+z)(x+y+\bar{z})(x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+\bar{z})$

27. Ans: (a)

Sol : M(a,b,c) = ab+bc+ca

K map for M(a,b,c)

a \ bc	00	01	11	10
0	0	0	1	0
1	0	1	1	1

Minterms are 3, 5, 6, 7

K map for M(a,b,c)

a \ bc	00	01	11	10
0	1	1	0	1
1	1	0	0	0

Minterms are 0, 1, 2, 4

$\overline{m(a,b,c)} = \bar b\bar c + \bar a\bar b + \bar a\bar c = x$

C = Z

M(X, Y, Z) = XY+YZ+ZX

$= (\bar b\bar c + \bar a\bar b + \bar a\bar c)(ab + b\bar c + a\bar c) + (\bar b\bar c + \bar a\bar b + \bar a\bar c)(c)$

$= a\bar b\bar c + \bar a b\bar c + abc + \bar a\bar b c$

$= \bar c[a\bar b + \bar a b] + c[ab + \bar a\bar b]$

$= \bar q p + q\bar p$

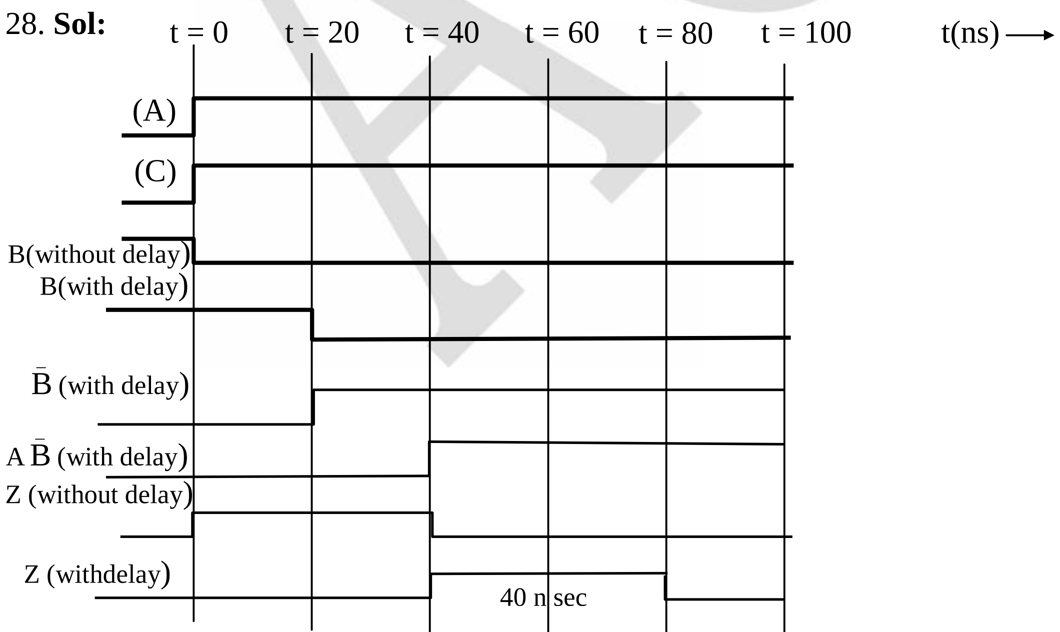
$= p \oplus q$; where q = c , $p = a \oplus b$

$\therefore$ M (x, y, z) = $a \oplus b \oplus c$

$\therefore$ 3 input X-OR realizes the equation

$M\left[\overline{M(a,b,c)}, M(a,b,\bar c), c\right]$

28. **Sol:**



$\therefore$ Z is 1 for 40 nsec

29.Ans: 7

Sol: 16kb memory is to be designed with no of rows = no. of columns

Let no of rows = no. of columns = x.

$16 \times 1024 = 16384 = x^2$

$\Rightarrow x = 128$

i.e. 128 rows and 128 columns.

To address 128 row let no. of decoder address lines = n.

$2^n = 128 \Rightarrow n = 7$

30. **Ans: 0.937 V**

Sol: 0001 $\rightarrow$ 0.0625 V corresponds to step size.

$\therefore f_{so} = (2^n - 1) \times \text{step size}$

$= 15 \times 0.0625$

$= 0.9375$

31. Ans: 8.885

Sol: $\vec{E} = 2\cos\left(10^8 t - \frac{Z}{\sqrt{2}}\right)$

$\beta = \frac{1}{\sqrt{2}}$ rad/m; $\omega = 10^8$ rad/s

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\frac{1}{\sqrt{2}}} = 8.885\text{m}$$

32. Ans: (a)

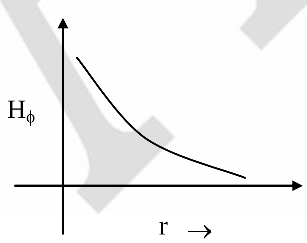
Sol: $H_\phi = \frac{I}{2\pi r}\hat{a}_\phi$

r = radial distance

I = current in the filament

$$H_\phi = \frac{I}{2\pi}\left(\frac{1}{r}\right)a_\phi$$

$$H_\phi = K\left(\frac{1}{r}\right)$$



$$H_\phi \alpha \left(\frac{1}{r}\right)$$

33. Ans: (c)

Sol: $H_z = \cos m\frac{\pi}{a}x \ \cos n\frac{\pi}{b}y \ \cos\beta z$

$$\frac{m}{0.08} = 25 \Rightarrow m = 2$$

$$\frac{n}{0.033} = 30.3 \Rightarrow n = 1$$

$\therefore$ H_z is existing so TE_{21}

34. Ans: 53.31

Sol: $E = 24\pi \cos(\omega t - \beta x)$ v/m $\hat{a}_z$

$$P_{avg} = \oint \vec{\delta}\, ds$$

Surface is given as x+y =1

$$\therefore \vec{d}s = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$$

Poynting vector

$$\vec{\delta} = \frac{E_0^2}{2\eta}\hat{a}_x$$

$$\therefore P_{avg} = \frac{(24\pi)^2}{2\times 120\pi} \times \frac{1}{\sqrt{2}} \times 10 \times 10 \times 10^{-4}$$

= 53.31 mW

36. Ans:(d)

Sol: $x(t) * h(t) = y(t)$

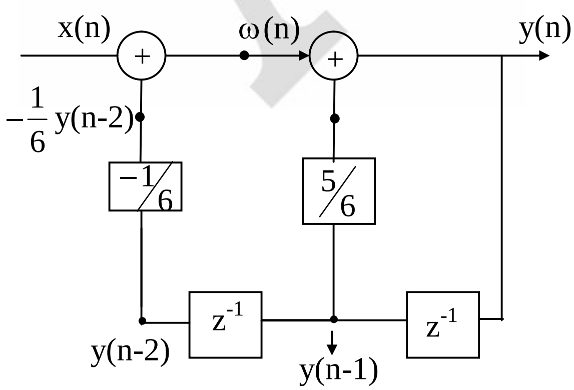
$x(-t) * h(-t) = y(-t)$

$x(t) * \delta(t - t_o) = x(t - t_o)$

$x(-t) * \delta(-t - t_o) = x(-t - t_o)$

38. Ans:(c)

Sol:



$\omega(n) = x(n) - \frac{1}{6}y(n-1)$ ------- (1)

$y(n) = \omega(n) - \frac{5}{6}y(n-1) \quad \therefore y(n) = x(n) - \frac{1}{6}y(n-2) + \frac{5}{6}y(n-1)$

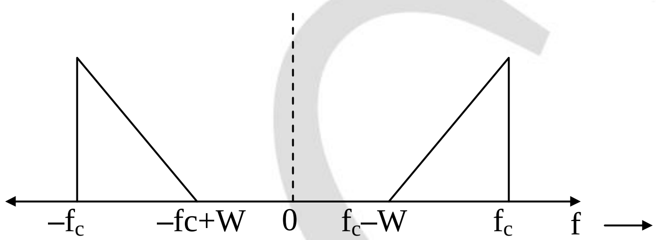
$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1+\frac{1}{6}z^{-2} - \frac{5}{6}z^{-1}} \qquad = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

$\therefore$ poles at $z = \frac{1}{3}, \frac{1}{2}$

41. Ans: (c)

Sol: $x(t) = m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t$.

The given signal is the lower sideband of a SSB – SC.



The signal is clearly a bandpass signal.

43. Ans: (a)

Sol: Δ = step size = 0.1 V

f_s = sampling rate = 20,000 samples/sec

f_m = message signal = 2000 Hz

A_m maximum amplitude of message signal to avoid slope overload distortion

$\frac{\Delta}{T_s} > 2\pi f_m A_m$;

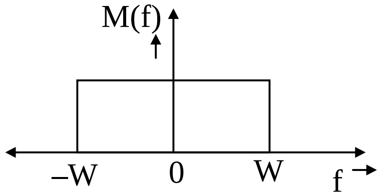
$0.1 \times 20{,}000 > 2\pi \times 2000 \times A_m \quad \left(\because f_s = \frac{1}{T_s}\right)$

$A_m < \frac{0.1 \times 20000}{2\pi \times 2000}$

$A_m = \frac{1}{2\pi}$

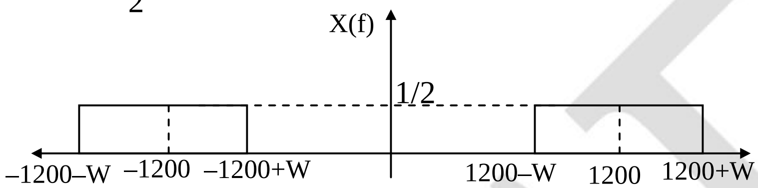
44. Ans: 350

Sol: Let M(f) be as shown in figure



x(t) = m(t) cos (2400 πt)

$X(f) = \frac{1}{2}[M(f - f_c) + M(f + f_c)]$; f_c = 1200 Hz

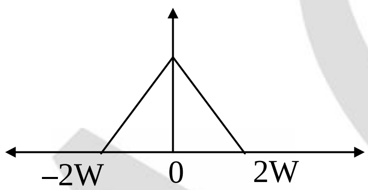


---------- (1)

Y(f) = X(f) ∗ X(f) + 10 X(f)

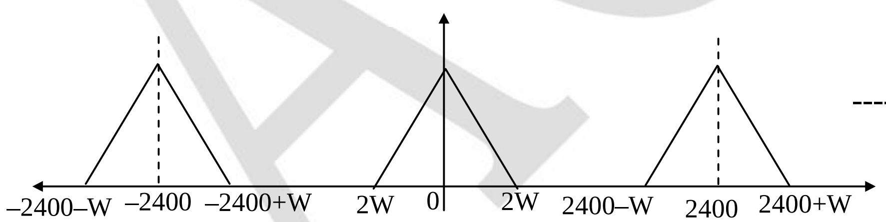
$x^2(t) = \frac{m^2(t)}{2}[1 + \cos 4\pi f_c t]$

Spectrum for $m^2(t)$ is



------------ (2)

Spectrum for $m^2(t)$ [1 + cos $4\pi f_c t$] is



------------ (3)

For the output to be z(t) = 10x(t)

From diagram (1), (2), (3) we have

1200 – W ≥ 700 & 1200 + W ≤ 1700

W ≤ 500

⇒ W > 300 Hz

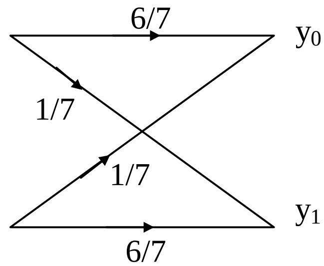
2W ≤ 700

W ≤ 350

W_{max} = 350 Hz

46 .Ans: 0.4

Sol: $p(x_0) = 0.2$
$p(x_1) = 0.8$
$p(y_0/x_0) = p(y_1/x_1) = 6/7$
$p(y_0/x_1) = p(y_1/x_0) = 1/7$
$p(x_1/y_0) = ?$



$$p(x_1/y_0) = \frac{p(y_0/x_1)p(x_1)}{p(y_0)} = \frac{p(y_0/x_1)p(x_1)}{p(y_0/x_1)p(x_1)+p(y_0/x_0)p(x_0)} = \frac{\frac{1}{7}\times 0.8}{\frac{1}{7}\times 0.8+\frac{6}{7}\times 0.2}$$

$p(x_1/y_0) = 0.4$

47. The solution of the differential equation.

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0,\quad x(0)=x'(0)=1$$

Sol : Auxiliary equation $D^2+2D+1=0$

$$(D+1)^2 = 0$$

$$D = -1,\ -1$$

The solution is $x = (C_1+C_2t)e^{-t}$ --------(1)

$x' = -(C_1+C_2t)e^{-t} + C_2e^{-t}$ --------(2)

$t=0,\ x=1,\ (1) \Rightarrow\ 1 = C_1$

$t=0,\ x'=1,\ (1) \Rightarrow\ 1 = -C_1+C_2\ \Rightarrow C_2 = 2$

solution is $x(t) = (1+2t)\,e^{-t}$

48.Ans: 17

Sol: $AX = \lambda X$

$$\begin{bmatrix} 4 & 1 & 2 \\ P & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \lambda \\ 2\lambda \\ 3\lambda \end{bmatrix}$$

$\lambda = 12$

$P+4+3 = 2\lambda = 24$

$P = 17$

49.Ans: (b)

Sol: $f(x) = 1-x^2+x^3$ in $[-1, 1]$

$f'(x) = -2x + 3x^2$

from the mean value theorem,

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(1)-f(-1)}{2}$$

$$\Rightarrow -2C + 3C^2 = \frac{(1-1+1)-(1-1-1)}{2} = 1$$

$\Rightarrow 3C^2 - 2C - 1 = 0$

$C = 1, \ -\frac{1}{3}$

$\therefore\ C = -\frac{1}{3} \in (-1, 1)$

C = 1 does not lie in (-1, 1)

50.Ans: (c)

Sol: Given

$$\overline{P} = (x^3y)\hat{a}_x + (-x^2y^2)\hat{a}_y + (-x^2yz)\hat{a}_z$$

(i) div $\overline{P} = \Delta.\overline{P}$

$$= \frac{\partial}{\partial x}(x^3y) + \frac{\partial}{\partial y}(-x^2y^2) + \frac{\partial}{\partial z}(-x^2yz)$$

$= 3x^2y - 2x^2y + (-x^2y)$

$= 0$

$\therefore\ \overline{P}$ is a solenoidal vector

(ii) curl $\overline{P} = \Delta \times \overline{P}$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y & -x^2y^2 & -x^2yz \end{vmatrix}$$

$= (-x^2z - 0)\,\hat{a}_x - (-2xyz - 0)\,\hat{a}_y + (-2xy^2 - x^3)\,\hat{a}_z$

$\neq 0$

$\therefore\ \overline{P}$ is a rotational vector

Hence the given vector $\overline{P}$ is solenoidal but not irrotational

51.Ans: 1

Sol Given ellipse equation is

$x^2 + 3y^2 = 1$ ------------(1)

let the area of the rectangle inscribed in the ellipse be $A = 2x \times 2y$

Let it be $f(x,y) = 4xy = 4x.\frac{\sqrt{1-x^2}}{2}$

$f(x) = 2x\sqrt{1-x^2}$

$f'(x) = 0$

$$\frac{2x \times -2x}{2\sqrt{1-x^2}} + 2\sqrt{1-x^2} = 0$$

$$\frac{-2x^2}{\sqrt{1-x^2}} + 2\sqrt{1-x^2} = 0$$

$-2x^2 + 2(1-x^2) = 0$

$-4x^2 + 2 = 0$

$4x^2 = 2$

$x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Area $= 4xy = 2x\sqrt{1-x^2} = 2\frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{2}}$

$= 1$

52.Ans:(b)

Sol : $f(x) = e^{-x}(1+x+x^2)$

$f'(x) = -e^{-x}[1+x+x^2] + e^{-x}[2x+1]$

$f'(x) = 0 \Rightarrow 2x+1-1-x-x^2 = 0.$

$\Rightarrow x = 0/1$

$f''(x) = e^{-x}(1+x+x^2) - e^{-x}(1+2x) - e^{-x}(2x+1) + e^{-x}(2)$

$= e^{-x}[1+x+x^2-1-2x-2x-1+2]$

$= e^{-x}(x^2-3x+1)$

At x=1

$= f''(1) = e^{-1}[1-3+1]$

$= -0.3678$

$f''(1) < 0 \Rightarrow$ matrix at x = 1

from the options,

option (b) satisfies the above analysis.

53. Sol : (a) $f(z) = \frac{1}{z^2-1} = \frac{\left(\frac{1}{z}+1\right)}{z-1}$ has a singular point at z=1

$\text{Res}(f(z) : z=1) = \left(\frac{1}{z+1}\right)_{z=1} = \frac{1}{1+1} = \frac{1}{2}$

$\therefore$ option (a) is a correct statement

(b) $I = \oint_C f(z)dz = \oint_C z^2 dz$

Here $f(z) = z^2$ is analytic function everywhere

$\therefore \oint_C z^2 dz = 0$ and option (b) is a correct statement

(c) $f(z) = \bar{z} = x - iy$

$\Rightarrow u + iv = f(z) = x - iy$

$\Rightarrow u = x$ and $v = -y$

$\Rightarrow u_x = 1 \quad v = 0$

$u_y = 0 \quad v_y = -1$

Here $u_x = 1 \neq v_y = -1$

i.e . one of the C.R. equation is not satisfied.

$\therefore f(z) = \bar{z}$ is not analytic function

Hence the given statement that $f(z) = \bar{z}$ is analytic, is a wrong statement.

54. Ans:C

Sol: As events A &B are independent

$\therefore P(A\cap B) = P(A)\,P(B)$

$P(B/A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{P(A)P(B)}{P(A)}$

$= P(B)$

$P(\bar{A}\cap \bar{B}) = P(\bar{A})P(\bar{B})$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= [P(A) + P(B) - P(A)\,P(B)]$

55.Sol:

The condition for many solutions of the system AX = B is

$\rho(A) = \rho(A/B) < n = 3.$

Or

For many solutions, we can consider $|A| = 0$,

Because, the constrant 'k' is in coefficient matrix.

Now $\begin{vmatrix} -1 & 2 & -3 \\ 1 & 6 & 12 \\ 2 & -4 & 3k \end{vmatrix} = 0$

$\Rightarrow 24k - 48 = 0$

$\Rightarrow k = 2.$

$\therefore$ The system has many solutions for k = 2.

General Aptitude:

01.Ans: (b)

02.Ans: (a)

03.Ans:1.

Sol: $66 \quad 6 = \dfrac{66-6}{66+6} = \dfrac{60}{72} = \dfrac{5}{6}$

$66 \quad 6 = \dfrac{66+6}{66-6} = \dfrac{72}{60} = \dfrac{6}{5}$

$66 \quad 6 \rightarrow (66 \quad 6) = \dfrac{5}{6} \times \dfrac{6}{5} = 1$

04.Ans:2.72

Sol: $\log_b a = \dfrac{\log_e a}{\log_e b}$

$\therefore \log_x 5/7 = \frac{\log_e 5/7}{\log_e x} = -1/3$

$\therefore \log_e x = 1 \Rightarrow x = e$

$\Rightarrow x = 2.72$

05. Ans(b)

Sol: Total number of small cubes = 27

Total number of faces = $27 \times 6 = 162$

Each face of big cube shows 9 faces of small cubes

$\therefore$ Total number of faces visible = $9 \times 6 = 54$

$\therefore$ Total number of invisible faces = 162 – 54 = 108

$\frac{\text{Visible faces}}{\text{Invisible faces}} = \frac{54}{108} = \frac{1}{2}$

06. Ans : (b)

08. Ans : (b)

09. Ans : (a)