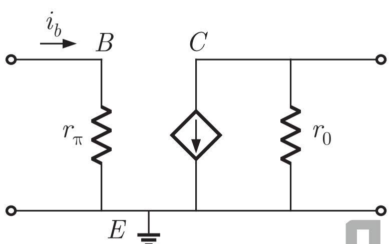
GATE EC 2012

Q. 1 - Q. 25 carry one mark each.

MCQ 1.1 The current i_b through the base of a silicon npn transistor is $1+0.1\cos(10000\pi t)$ mA At 300 K, the r_π in the small signal model of the transistor is



(A) $250\,\Omega$ (B) $27.5\,\Omega$

(C) $25\,\Omega$ (D) $22.5\,\Omega$

SOL 1.1 Option (C) is correct.

Given $i_b = 1+0.1\cos(1000\pi t)$ mA

So, $I_B =$ DC component of i_b

$= 1$ mA

In small signal model of the transistor

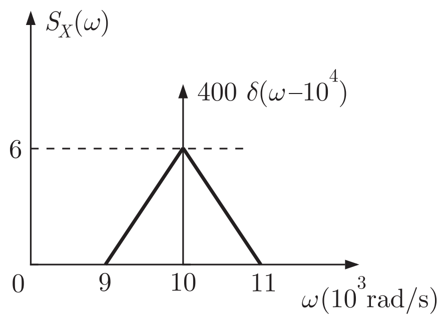
$$r_\pi = \frac{\beta V_T}{I_C} \qquad V_T \to \text{Thermal voltage}$$

$$= \frac{V_T}{I_C/\beta} = \frac{V_T}{I_B} \qquad \frac{I_C}{\beta} = I_B$$

$$= \frac{V_T}{I_B}$$

So, $$r_\pi = \frac{25\text{ mV}}{1\text{ mA}} = 25\,\Omega \qquad V_T = 25\text{ mV},\ I_B = 1\text{ mA}$$

MCQ 1.2 The power spectral density of a real process $X(t)$ for positive frequencies is shown below. The values of $E[X^2(t)]$ and $|E[X(t)]|$, respectively, are



(A) $6000/\pi, 0$ (B) $6400/\pi, 0$

(C) $6400/\pi,\ 20/(\pi\sqrt{2})$ (D) $6000/\pi,\ 20/(\pi\sqrt{2})$

SOL 1.2 Option (A) is correct.

The mean square value of a stationary process equals the total area under the graph of power spectral density, that is

$$E[X^2(t)] = \int_{-\infty}^{\infty} S_X(f)\, df$$

or,
$$E[X^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\, d\omega$$

or,
$$E[X^2(t)] = 2 \times \frac{1}{2\pi}\int_{0}^{\infty} S_X(\omega)\, d\omega \qquad \text{(Since the PSD is even)}$$

$$= \frac{1}{\pi}[\text{area under the triangle} + \text{integration of delta function}]$$

$$= \frac{1}{\pi}\left[2\left(\frac{1}{2} \times 1 \times 10^3 \times 6\right) + 400\right]$$

$$= \frac{1}{\pi}[6000 + 400]$$

$$= \frac{6400}{\pi}$$

$|E[X(t)]|$ is the absolute value of mean of signal $X(t)$ which is also equal to value of $X(\omega)$ at $(\omega = 0)$.

From given PSD

$$S_X(\omega)\big|_{\omega=0} = 0$$

$$S_X(\omega) = |X(\omega)|^2 = 0$$

$$|X(\omega)|^2_{\omega=0} = 0$$

$$|X(\omega)|_{\omega=0} = 0$$

MCQ 1.3 In a baseband communications link, frequencies upto 3500 Hz are used for signaling. Using a raised cosine pulse with 75% excess bandwidth and for no inter-symbol interference, the maximum possible signaling rate in symbols per second is

(A) 1750 (B) 2625

(C) 4000 (D) 5250

SOL 1.3 Option (C) is correct.

For raised cosine spectrum transmission bandwidth is given as

$$B_T = W(1+\alpha) \qquad \alpha \rightarrow \text{Roll of factor}$$

$$B_T = \frac{R_b}{2}(1+\alpha) \qquad R_b \rightarrow \text{Maximum signaling rate}$$

$$3500 = \frac{R_b}{2}(1+0.75)$$

$$R_b = \frac{3500 \times 2}{1.75} = 4000$$

MCQ 1.4 A plane wave propagating in air with $\boldsymbol{E} = (8\boldsymbol{a}_x + 6\boldsymbol{a}_y + 5\boldsymbol{a}_z)\, e^{j(\omega t + 3x - 4y)}$ V/m is incident on a perfectly conducting slab positioned at $x \leq 0$. The $\boldsymbol{E}$ field of the reflected wave is

(A) $(-8\boldsymbol{a}_x - 6\boldsymbol{a}_y - 5\boldsymbol{a}_z)\, e^{j(\omega t + 3x + 4y)}$ V/m (B) $(-8\boldsymbol{a}_x + 6\boldsymbol{a}_y - 5\boldsymbol{a}_z)\, e^{j(\omega t + 3x + 4y)}$ V/m

(C) $(-8\boldsymbol{a}_x - 6\boldsymbol{a}_y - 5\boldsymbol{a}_z)\, e^{j(\omega t - 3x - 4y)}$ V/m (D) $(-8\boldsymbol{a}_x + 6\boldsymbol{a}_y - 5\boldsymbol{a}_z)\, e^{j(\omega t - 3x - 4y)}$ V/m

SOL 1.4 Option (C) is correct.

Electric field of the propagating wave in free space is given as

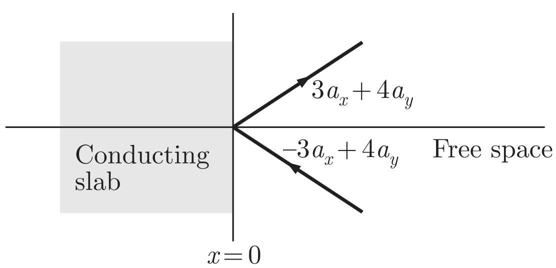
$$\boldsymbol{E}_i = (8\boldsymbol{a}_x + 6\boldsymbol{a}_y + 5\boldsymbol{a}_z)\, e^{j(\omega t + 3x - 4y)} \text{ V/m}$$

So, it is clear that wave is propagating in the direction $(-3\boldsymbol{a}_x + 4\boldsymbol{a}_y)$.

Since, the wave is incident on a perfectly conducting slab at $x = 0$. So, the reflection coefficient will be equal to -1.

i.e. $$E_{r_0} = (-1)\, E_{i_0}$$
$$= -8\boldsymbol{a}_x - 6\boldsymbol{a}_y - 5\boldsymbol{a}_z$$

Again, the reflected wave will be as shown in figure.



i.e. the reflected wave will be in direction $3a_x + 4a_y$. Thus, the electric field of the reflected wave will be.

$$\boldsymbol{E}_x = (-8\boldsymbol{a}_x - 6\boldsymbol{a}_y - 5\boldsymbol{a}_z)\, e^{j(\omega t - 3x - 4y)} \text{ V/m}$$

MCQ 1.5 The electric field of a uniform plane electromagnetic wave in free space, along the positive x direction is given by $\boldsymbol{E} = 10(\boldsymbol{a}_y + j\boldsymbol{a}_z)\, e^{-j25x}$. The frequency and polarization of the wave, respectively, are

(A) 1.2 GHz and left circular (B) 4 Hz and left circular

(C) 1.2 GHz and right circular (D) 4 Hz and right circular

SOL 1.5 Option (A) is correct.

The field in circular polarization is found to be

$$E_s = E_0(\boldsymbol{a}_y \pm j\boldsymbol{a}_z)\,e^{-j\beta x} \text{ propagating in +ve } x\text{-direction.}$$

where, plus sign is used for left circular polarization and minus sign for right circular polarization. So, the given problem has left circular polarization.

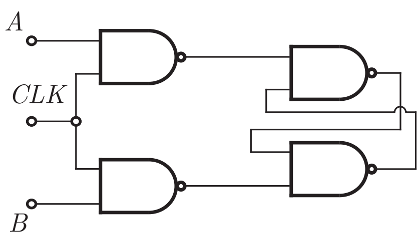
$$\beta = 25 = \frac{\omega}{c}$$

$$25 = \frac{2\pi f}{c}$$

$$f = \frac{25 \times c}{2\pi} = \frac{25 \times 3 \times 10^8}{2 \times 3.14}$$

$$= 1.2\,\text{GHz}$$

MCQ 1.6 Consider the given circuit



In this circuit, the race around

(A) does not occur

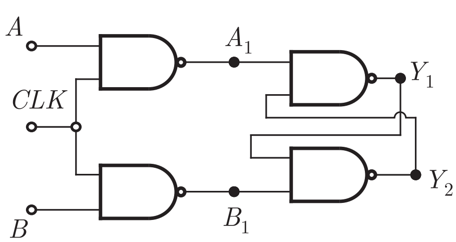
(B) occur when $CLK = 0$

(C) occur when $CLK = 1$ and $A = B = 1$

(D) occur when $CLK = 1$ and $A = B = 0$

SOL 1.6 Option (A) is correct.

The given circuit is



Condition for the race-around

It occurs when the output of the circuit (Y_1, Y_2) oscillates between '0' and '1' checking it from the options.

1. Option (A): When $CLK = 0$

Output of the NAND gate will be $A_1 = B_1 = \overline{0} = 1$. Due to these input to the next NAND gate, $Y_2 = \overline{Y_1 \cdot 1} = \overline{Y_1}$ and $Y_1 = \overline{Y_2 \cdot 1} = \overline{Y_2}$.

If $Y_1 = 0$, $\quad Y_2 = \overline{Y_1} = 1$ and it will remain the same and doesn't oscillate.

If $Y_2 = 0$, $\quad Y_1 = \overline{Y_2} = 1$ and it will also remain the same for the clock period. So,

it won't oscillate for $CLK = 0$.
So, here race around doesn't occur for the condition $CLK = 0$.
2. Option (C): When $CLK = 1$, $A = B = 1$

$$A_1 = B_1 = 0 \text{ and so } Y_1 = Y_2 = 1$$

And it will remain same for the clock period. So race around doesn't occur for the condition.
3. Option (D): When $CLK = 1$, $A = B = 0$
So, $\quad A_1 = B_1 = 1$
And again as described for Option (B) race around doesn't occur for the condition.
So, Option (A) will be correct.

MCQ 1.7 The output Y of a 2-bit comparator is logic 1 whenever the 2-bit input A is greater than the 2-bit input B. The number of combinations for which the output is logic 1, is

(A) 4 (B) 6
(C) 8 (D) 10

SOL 1.7 Option (B) is correct.

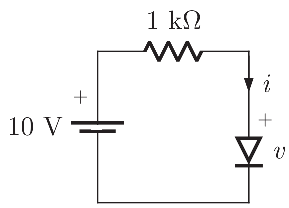
$$Y = 1, \text{ when } A > B$$
$$A = a_1 a_0, \ B = b_1 b_0$$

a_1	a_0	b_1	b_0	Y
0	1	0	0	1
1	0	0	0	1
1	0	0	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1

Total combination $= 6$

MCQ 1.8 The i-v characteristics of the diode in the circuit given below are

$$i = \begin{cases} \dfrac{v - 0.7}{500} \text{ A}, & v \geq 0.7 \text{ V} \\ 0 \text{ A} & v < 0.7 \text{ V} \end{cases}$$



The current in the circuit is

(A) 10 mA (B) 9.3 mA

(C) 6.67 mA (D) 6.2 mA

SOL 1.8 Option (D) is correct.

Let $v > 0.7$ V and diode is forward biased. By applying Kirchoff's voltage law

$$10 - i \times 1\text{k} - v = 0$$

$$10 - \left[\frac{v - 0.7}{500}\right](1000) - v = 0$$

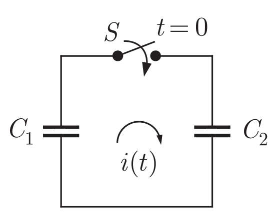
$$10 - (v - 0.7) \times 2 - v = 0$$

$$10 - 3v + 1.4 = 0$$

$$v = \frac{11.4}{3} = 3.8\text{ V} > 0.7 \qquad \text{(Assumption is true)}$$

So, $$i = \frac{v - 0.7}{500} = \frac{3.8 - 0.7}{500} = 6.2\text{ mA}$$

MCQ 1.9 In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at $t = 0$. The current $i(t)$ for all t is

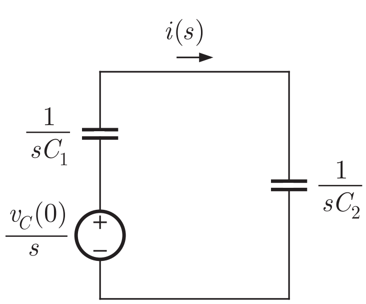


(A) zero (B) a step function

(C) an exponentially decaying function (D) an impulse function

SOL 1.9 Option (D) is correct.

The s-domain equivalent circuit is shown as below.



$$I(s) = \frac{v_c(0)/s}{\frac{1}{C_1 s} + \frac{1}{C_2 s}} = \frac{v_c(0)}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$I(s) = \left(\frac{C_1 C_2}{C_1 + C_2}\right)(12\text{ V}) \qquad v_C(0) = 12\text{ V}$$

$$I(s) = 12C_{eq}$$

Taking inverse Laplace transform for the current in time domain,

$$i(t) = 12C_{eq}\delta(t) \qquad \text{(Impulse)}$$

MCQ 1.10 The average power delivered to an impedance $(4 - j3)\,\Omega$ by a current $5\cos(100\pi t + 100)$ A is

(A) 44.2 W (B) 50 W

(C) 62.5 W (D) 125 W

SOL 1.10 Option (B) is correct.

In phasor form

$$Z = 4 - j3$$

$$Z = 5\angle{-36.86^\circ}\,\Omega$$

$$\boldsymbol{I} = 5\angle{100^\circ}\text{ A}$$

Average power delivered.

$$P_{avg.} = \frac{1}{2}|\boldsymbol{I}|^2 Z\cos\theta$$

$$= \frac{1}{2} \times 25 \times 5\cos 36.86^\circ$$

$$= 50\text{ W}$$

Alternate method:

$$Z = (4 - j3)\,\Omega$$

$$I = 5\cos(100\pi t + 100)\text{ A}$$

$$P_{avg} = \frac{1}{2}\text{Re}\left\{|I|^2 Z\right\}$$

$$= \frac{1}{2} \times \text{Re}\{(5)^2 \times (4 - j3)\}$$

$$= \frac{1}{2} \times 100 = 50\text{ W}$$

MCQ 1.11 The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2 + s + 1}$. The unilateral Laplace transform of $tf(t)$ is

(A) $-\frac{s}{(s^2 + s + 1)^2}$ (B) $-\frac{2s + 1}{(s^2 + s + 1)^2}$

(C) $\frac{s}{(s^2 + s + 1)^2}$ (D) $\frac{2s + 1}{(s^2 + s + 1)^2}$

SOL 1.11 Option (D) is correct.

Using s-domain differentiation property of Laplace transform.

If $$f(t) \xleftrightarrow{\mathcal{L}} F(s)$$

$$tf(t) \xleftrightarrow{\mathcal{L}} -\frac{dF(s)}{ds}$$

So, $$\mathcal{L}[tf(t)] = \frac{-d}{ds}\left[\frac{1}{s^2+s+1}\right]$$

$$= \frac{2s+1}{(s^2+s+1)^2}$$

MCQ 1.12 With initial condition $x(1) = 0.5$, the solution of the differential equation $t\frac{dx}{dt} + x = t$, is

(A) $x = t - \frac{1}{2}$

(B) $x = t^2 - \frac{1}{2}$

(C) $x = \frac{t^2}{2}$

(D) $x = \frac{t}{2}$

SOL 1.12 Option (D) is correct.

$$t\frac{dx}{dt} + x = t$$

$$\frac{dx}{dt} + \frac{x}{t} = 1$$

$$\frac{dx}{dt} + Px = Q \text{ (General form)}$$

Integrating factor, $$IF = e^{\int Pdt} = e^{\int \frac{1}{t}dt} = e^{\ln t} = t$$

Solution has the form

$$x \times IF = \int (Q \times IF)\,dt + C$$

$$x \times t = \int (1)(t)\,dt + C$$

$$xt = \frac{t^2}{2} + C$$

Taking the initial condition

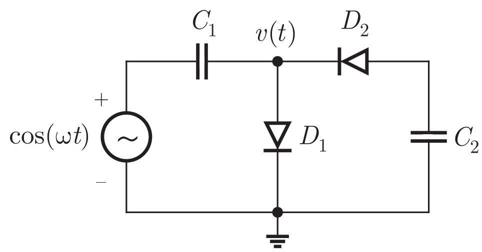
$$x(1) = 0.5$$

$$0.5 = \frac{1}{2} + C$$

$$C = 0$$

So, $$xt = \frac{t^2}{2} \Rightarrow x = \frac{t}{2}$$

MCQ 1.13 The diodes and capacitors in the circuit shown are ideal. The voltage $v(t)$ across the diode D_1 is

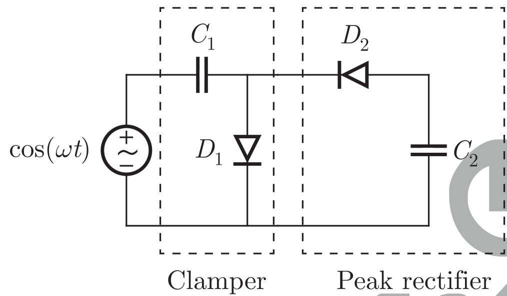


(A) $\cos(\omega t) - 1$ (B) $\sin(\omega t)$

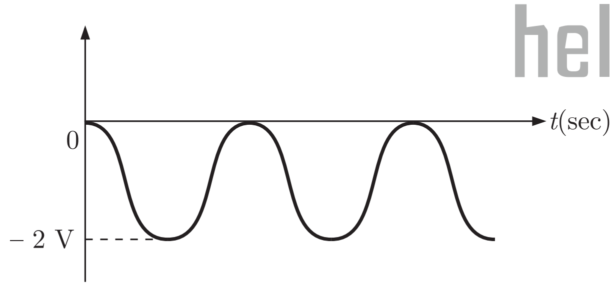
(C) $1 - \cos(\omega t)$ (D) $1 - \sin(\omega t)$

SOL 1.13 Option (A) is correct.

The circuit composed of a clamper and a peak rectifier as shown.



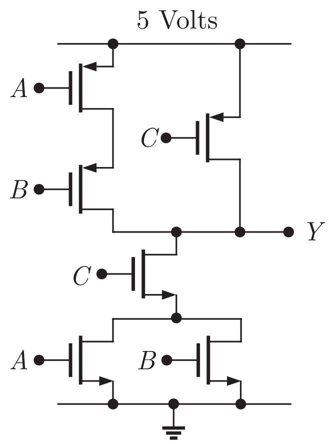
Clamper clamps the voltage to zero voltage, as shown



The peak rectifier adds +1 V to peak voltage, so overall peak voltage lowers down by −1 volt.

So, $$v_o = \cos\omega t - 1$$

MCQ 1.14 In the circuit shown



(A) $Y = \overline{A}\,\overline{B} + \overline{C}$
(B) $Y = (A + B)\,C$
(C) $Y = (\overline{A} + \overline{B})\,\overline{C}$
(D) $Y = AB + C$

SOL 1.14 Option (A) is correct.

Parallel connection of $MOS \Rightarrow OR$ operation

Series connection of $MOS \Rightarrow AND$ operation

The pull-up network acts as an inverter. From pull down network we write

$$Y = \overline{(A+B)\,C}$$
$$Y = \overline{(A+B)} + \overline{C}$$
$$= \overline{A}\,\overline{B} + \overline{C}$$

MCQ 1.15 A source alphabet consists of N symbols with the probability of the first two symbols being the same. A source encoder increases the probability of the first symbol by a small amount ε and decreases that of the second by ε. After encoding, the entropy of the source

(A) increases
(B) remains the same
(C) increases only if $N = 2$
(D) decreases

SOL 1.15 Option (D) is correct.

Entropy function of a discrete memory less system is given as

$$H = \sum_{k=0}^{N-1} P_k \log\left(\frac{1}{P_k}\right)$$

where P_k is probability of symbol S_k.

For first two symbols probability is same, so

$$H = P_1 \log\left(\frac{1}{P_1}\right) + P_2 \log\left(\frac{1}{P_2}\right) + \sum_{k=3}^{N-1} P_k \log\left(\frac{1}{P_k}\right)$$
$$= -\left(P_1 \log P_1 + P_2 \log P_2 + \sum_{k=3}^{N-1} P_k \log P_k\right)$$
$$= -\left(2P \log P + \sum_{k=3}^{N-1} P_k \log P_k\right) \qquad (P_1 = P_2 = P)$$

Now, $P_1 = P + \varepsilon,\ P_2 = P - \varepsilon$

So, $$H' = -\left[(P+\varepsilon)\log(P+\varepsilon) + (P-\varepsilon)\log(P-\varepsilon) + \sum_{k=3}^{N-1} P_k \log P_k\right]$$

By comparing, $H' < H$

Entropy of source decreases.

MCQ 1.16 A coaxial-cable with an inner diameter of 1 mm and outer diameter of 2.4 mm is filled with a dielectric of relative permittivity 10.89. Given $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m, the characteristic impedance of the cable is

(A) $330\,\Omega$ (B) $100\,\Omega$

(C) $143.3\,\Omega$ (D) $43.4\,\Omega$

SOL 1.16 Option (B) is correct.

Characteristic impedance.

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{b}{a}\right)$$

$b \rightarrow$ outer diameter

$a \rightarrow$ inner diameter

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}\ln\left(\frac{b}{a}\right)$$

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 36\pi}{10^{-9} \times 10.89}}\ln\left(\frac{2.4}{1}\right)$$

$$= 100\,\Omega$$

MCQ 1.17 The radiation pattern of an antenna in spherical co-ordinates is given by

$$F(\theta) = \cos^4\theta; \quad 0 \le \theta \le \pi/2$$

The directivity of the antenna is

(A) 10 dB (B) 12.6 dB

(C) 11.5 dB (D) 18 dB

SOL 1.17 Option (A) is correct.

The directivity is defined as

$$D = \frac{F_{\max}}{F_{avg}}$$

$$F_{\max} = 1$$

$$F_{avg} = \frac{1}{4\pi}\int F(\theta,\phi)\,d\Omega$$

$$= \frac{1}{4\pi}\left[\int_0^{2\pi}\int_0^{2\pi} F(\theta,\phi)\sin\theta d\theta\, d\phi\right]$$

$$= \frac{1}{4\pi}\left[\int_0^{2\pi}\int_0^{\pi/2} \cos^4\theta \sin\theta d\theta d\phi\right]$$

$$= \frac{1}{4\pi}\left[2\pi\left(-\frac{\cos^5\theta}{5}\right)\right]_0^{\pi/2}$$

$$= \frac{1}{4\pi} \times 2\pi\left[-0 + \frac{1}{5}\right]$$

$$= \frac{1}{4\pi} \times \frac{2\pi}{5} = \frac{1}{10}$$

$$D = \frac{1}{10} = 10$$

or, $D(\text{in dB}) = 10\log 10 = 10\,\text{dB}$

MCQ 1.18 If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its z-transform in the z-plane will be

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{1}{3} < |z| < \frac{1}{2}$

(C) $\frac{1}{2} < |z| < 3$ (D) $\frac{1}{3} < |z|$

SOL 1.18 Option (C) is correct.

$$x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1] - \left(\frac{1}{2}\right)^n u(n)$$

Taking z-transform

$$X[z] = \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^{-n} z^{-n} u[-n-1] - \sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} u[n]$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1}\left(\frac{1}{3}\right)^{-n} z^{-n} - \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}$$

$$= \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n}_{\text{I}} + \underbrace{\sum_{m=1}^{\infty}\left(\frac{1}{3}z\right)^m}_{\text{II}} - \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n}_{\text{III}} \qquad \text{Taking } m = -n$$

Series I converges if $\left|\frac{1}{3z}\right| < 1$ or $|z| > \frac{1}{3}$

Series II converges if $\left|\frac{1}{3}z\right| < 1$ or $|z| < 3$

Series III converges if $\left|\frac{1}{2z}\right| < 1$ or $|z| > \frac{1}{2}$

Region of convergence of $X(z)$ will be intersection of above three

So, ROC : $\frac{1}{2} < |z| < 3$

MCQ 1.19 In the sum of products function $f(X, Y, Z) = \sum(2,3,4,5)$, the prime implicants are

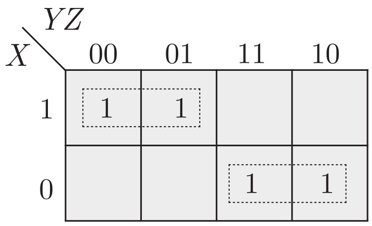
(A) $\overline{X}Y$, $X\overline{Y}$ (B) $\overline{X}Y$, $X\,\overline{Y}\,\overline{Z}$, $X\overline{Y}Z$

(C) $\overline{X}Y\overline{Z}$, $\overline{X}YZ$, $X\overline{Y}$

(D) $\overline{X}Y\overline{Z}$, $\overline{X}YZ$, $X\overline{Y}\,\overline{Z}$, $X\overline{Y}Z$

SOL 1.19 Option (A) is correct.

Prime implicants are the terms that we get by solving K-map



$$F = \underbrace{X\overline{Y} + \overline{X}Y}_{\text{prime implicants}}$$

MCQ 1.20 A system with transfer function

$$G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

is excited by $\sin(\omega t)$. The steady-state output of the system is zero at

(A) $\omega = 1\,\text{rad/s}$

(B) $\omega = 2\,\text{rad/s}$

(C) $\omega = 3\,\text{rad/s}$

(D) $\omega = 4\,\text{rad/s}$

SOL 1.20 Option (C) is correct.

$$G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

$$G(j\omega) = \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+1)(j\omega+3)(j\omega+4)}$$

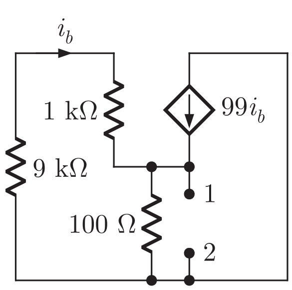
The steady state output will be zero if

$$|G(j\omega)| = 0$$

$$-\omega^2 + 9 = 0$$

$$\omega = 3\,\text{rad/s}$$

MCQ 1.21 The impedance looking into nodes 1 and 2 in the given circuit is



(A) $50\,\Omega$

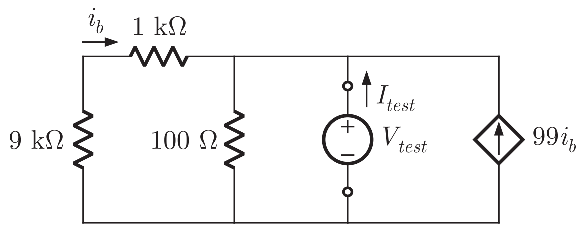
(B) $100\,\Omega$

(C) $5\,\text{k}\Omega$

(D) $10.1\,\text{k}\Omega$

SOL 1.21 Option (A) is correct.

We put a test source between terminal 1, 2 to obtain equivalent impedance



$$Z_{Th} = \frac{V_{test}}{I_{test}}$$

By applying KCL at top right node

$$\frac{V_{test}}{9\text{k}+1\text{k}} + \frac{V_{test}}{100} - 99I_b = I_{test}$$

$$\frac{V_{test}}{10\text{k}} + \frac{V_{test}}{100} - 99I_b = I_{test} \qquad ...(i)$$

But $$I_b = -\frac{V_{test}}{9k+1k} = -\frac{V_{test}}{10\text{k}}$$

Substituting I_b into equation (i), we have

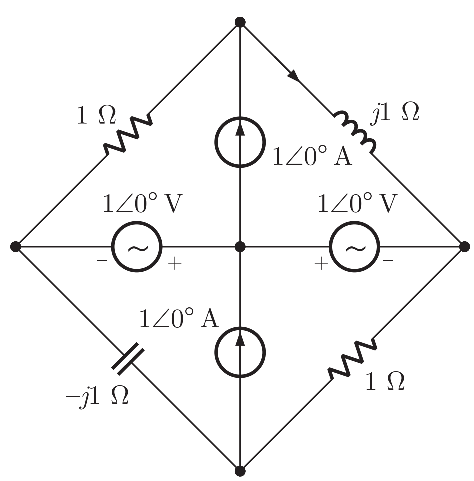
$$\frac{V_{test}}{10\text{k}} + \frac{V_{test}}{100} + \frac{99\,V_{test}}{10\text{k}} = I_{test}$$

$$\frac{100\,V_{test}}{10\times 10^3} + \frac{V_{test}}{100} = I_{test}$$

$$\frac{2\,V_{test}}{100} = I_{test}$$

$$Z_{Th} = \frac{V_{test}}{I_{test}} = 50\,\Omega$$

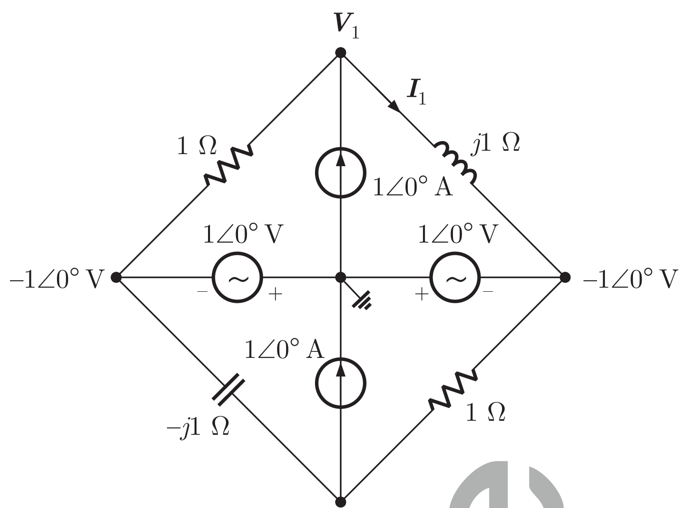
MCQ 1.22 In the circuit shown below, the current through the inductor is



(A) $\frac{2}{1+j}$ A (B) $\frac{-1}{1+j}$ A

(C) $\frac{1}{1+j}$ A (D) 0 A

SOL 1.22 Option (C) is correct.



Applying nodal analysis at top node.

$$\frac{V_1 + 1\angle 0^\circ}{1} + \frac{V_1 + 1\angle 0^\circ}{j1} = 1\angle 0^\circ$$

$$V_1(j1+1) + j1 + 1\angle 0^\circ = j1$$

$$V_1 = \frac{-1}{1+j1}$$

Current $$I_1 = \frac{V_1 + 1\angle 0^\circ}{j1} = \frac{-\frac{1}{1+j}+1}{j1}$$

$$= \frac{j}{(1+j)j} = \frac{1}{1+j}\text{ A}$$

MCQ 1.23 Given $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$.

If C is a counter clockwise path in the z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi j}\oint_C f(z)\,dz$ is

(A) -2 (B) -1

(C) 1 (D) 2

SOL 1.23 Option (C) is correct.

$$f(z) = \frac{1}{z+1} - \frac{2}{z+3}$$

$$\frac{1}{2\pi j}\oint_C f(z)\,dz = \text{sum of the residues of the poles which lie inside the given closed region.}$$

$$C \Rightarrow |z+1| = 1$$

Only pole $z = -1$ inside the circle, so residue at $z = -1$ is.

$$f(z) = \frac{-z+1}{(z+1)(z+3)}$$

$$= \lim_{z \to -1} \frac{(z+1)(-z+1)}{(z+1)(z+3)} = \frac{2}{2} = 1$$

So $\frac{1}{2\pi j} \oint_C f(z)\, dz = 1$

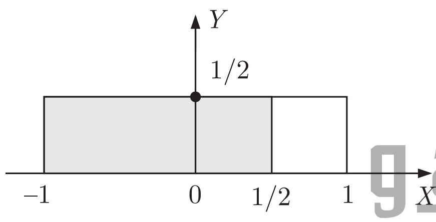
MCQ 1.24 Two independent random variables X and Y are uniformly distributed in the interval $[-1, 1]$. The probability that $\max[X, Y]$ is less than $1/2$ is

(A) 3/4 (B) 9/16

(C) 1/4 (D) 2/3

SOL 1.24 Option (B) is correct.

Probability density function of uniformly distributed variables X and Y is shown as



$$P\left\{[\max(x, y)] < \frac{1}{2}\right\}$$

Since X and Y are independent.

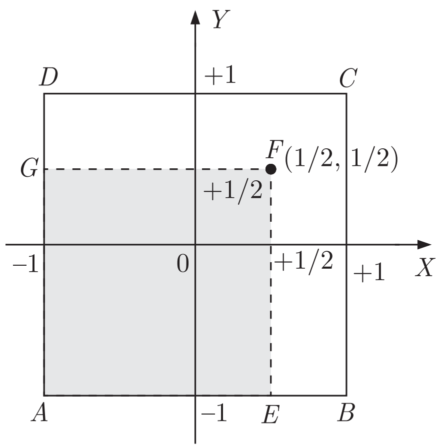
$$P\left\{[\max(x, y)] < \frac{1}{2}\right\} = P\left(X < \frac{1}{2}\right) P\left(Y < \frac{1}{2}\right)$$

$$P\left(X < \frac{1}{2}\right) = \text{shaded area} = \frac{3}{4}$$

Similarly for Y: $P\left(Y < \frac{1}{2}\right) = \frac{3}{4}$

So $P\left\{[\max(x, y)] < \frac{1}{2}\right\} = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

Alternate method:



From the given data since random variables X and Y lies in the interval $[-1,1]$ as from the figure X, Y lies in the region of the square $ABCD$.

Probability for $\max[X, Y] < 1/2$: The points for $\max[X, Y] < 1/2$ will be inside the region of square $AEFG$.

So, $$P\left\{\max[X, Y] < \frac{1}{2}\right\} = \frac{\text{Area of} \square AEFG}{\text{Area of square } ABCD}$$

$$= \frac{\frac{3}{2} \times \frac{3}{2}}{2 \times 2} = \frac{9}{16}$$

MCQ 1.25 If $x = \sqrt{-1}$, then the value of x^x is

(A) $e^{-\pi/2}$ (B) $e^{\pi/2}$

(C) x (D) 1

SOL 1.25 Option (A) is correct.

$$x = \sqrt{-1} = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

So, $$x = e^{i\frac{\pi}{2}}$$

$$x^x = \left(e^{i\frac{\pi}{2}}\right)^x \Rightarrow \left(e^{i\frac{\pi}{2}}\right)^i$$

$$= e^{-\frac{\pi}{2}}$$

Q. 26 to Q. 55 carry two marks each.

MCQ 1.26 The source of a silicon ($n_i = 10^{10}$ per cm^3) n-channel MOS transistor has an area of 1 sq µm and a depth of 1 µm. If the dopant density in the source is $10^{19}/\text{cm}^3$, the number of holes in the source region with the above volume is approximately

(A) 10^7 (B) 100

(C) 10 (D) 0

SOL 1.26 Option (D) is correct.

For the semiconductor

$$n_0 p_0 = n_i^2$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{10^{20}}{10^{19}} = 10 \text{ per cm}^3$$

Volume of given device,

$$\begin{aligned} V &= \text{Area} \times \text{depth} \\ &= 1\,\mu\text{m}^2 \times 1\,\mu\text{m} \\ &= 10^{-8}\text{cm}^2 \times 10^{-4}\text{cm} \\ &= 10^{-12}\,\text{cm}^3 \end{aligned}$$

So total no. of holes is,

$$\begin{aligned} p &= p_0 \times V \\ &= 10 \times 10^{-12} \\ &= 10^{-11} \end{aligned}$$

Which is approximately equal to zero.

MCQ 1.27 A BPSK scheme operating over an AWGN channel with noise power spectral density of $N_0/2$, uses equiprobable signals $s_1(t) = \sqrt{\frac{2E}{T}}\sin(\omega_c t)$ and $s_2(t) = -\sqrt{\frac{2E}{T}}\sin(\omega_c t)$ over the symbol interval $(0, T)$. If the local oscillator in a coherent receiver is ahead in phase by $45°$ with respect to the received signal, the probability of error in the resulting system is

(A) $Q\left(\sqrt{\frac{2E}{N_0}}\right)$ (B) $Q\left(\sqrt{\frac{E}{N_0}}\right)$

(C) $Q\left(\sqrt{\frac{E}{2N_0}}\right)$ (D) $Q\left(\sqrt{\frac{E}{4N_0}}\right)$

SOL 1.27 Option (B) is correct.

In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary system 1 and 0 respectively.

$$s_1(t) = \sqrt{\frac{2E}{T}}\sin\omega_c t$$

$$s_2(t) = -\sqrt{\frac{2E}{T}}\sin\omega_c t$$

where $0 \le t \le T$, E is the transmitted energy per bit.

General function of local oscillator

$$\phi_1(t) = \sqrt{\frac{2}{T}}\sin(\omega_c t),\ 0 \le t < T$$

But here local oscillator is ahead with $45°$. so,

$$\phi_1(t) = \sqrt{\frac{2}{T}}\sin(\omega_c t + 45°)$$

The coordinates of message points are

$$s_{11} = \int_0^T s_1(t)\,\phi_1(t)\,dt$$

$$= \int_0^T \sqrt{\frac{2E}{T}} \sin \omega_c t \sqrt{\frac{2}{T}} \sin (\omega_c t + 45^\circ)\, dt$$

$$= \sqrt{\frac{2E}{T}} \int_0^T \sin (\omega_c t) \sin (\omega_c t + 45^\circ)\, dt$$

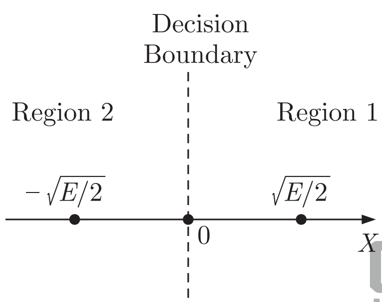
$$= \sqrt{\frac{2E}{T}} \sqrt{\frac{2}{T}} \int_0^T \frac{1}{2} [\sin 45^\circ + \sin (2\omega_c t + 45^\circ)]\, dt$$

$$= \frac{1}{T} \sqrt{E} \int_0^T \frac{1}{\sqrt{2}}\, dt + \underbrace{\frac{1}{T} \sqrt{E} \int_0^T \sin (2\omega_c t + 45^\circ)\, dt}_{0}$$

$$= \sqrt{\frac{E}{2}}$$

Similarly, $s_{21} = -\sqrt{\frac{E}{2}}$

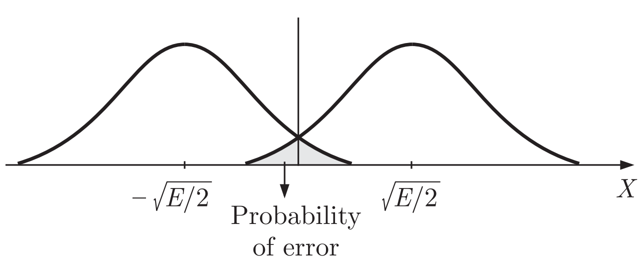
Signal space diagram



Now here the two message points are s_{11} and s_{21}.
The error at the receiver will be considered.
When : (i) s_{11} is transmitted and s_{21} received
(ii) s_{21} is transmitted and s_{11} received
So, probability for the 1st case will be as :

$$P\left(\frac{s_{21}\,\text{received}}{s_{11}\,\text{transmitted}}\right) = P(X < 0) \text{ (as shown in diagram)}$$
$$= P\left(\sqrt{E/2} + N < 0\right)$$
$$= P\left(N < -\sqrt{E/2}\right)$$

Taking the Gaussian distribution as shown below :



Mean of the Gaussian distribution $= \sqrt{E/2}$

Variance $= \frac{N_0}{2}$

Putting it in the probability function :

$$P\left(N < -\sqrt{\frac{E}{2}}\right) = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\frac{N_0}{2}}} e^{-\frac{\left(x+\sqrt{E/2}\right)^2}{2N_0/2}} dx$$

$$= \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(x+\sqrt{E/2}\right)^2}{N_0}} dx$$

Taking, $\dfrac{x+\sqrt{E/2}}{\sqrt{N_0/2}} = t$

$$dx = \sqrt{\frac{N_0}{2}}\, dt$$

So, $P\left(N < -\sqrt{E/2}\right) = \displaystyle\int_{\sqrt{E/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$

$$Q\left(\sqrt{\frac{E}{N_0}}\right)$$

where Q is error function.

Since symbols are equiprobable in the 2^{nd} case

So,

$$P\left(\frac{s_{11}\text{ received}}{s_{21}\text{ transmitted}}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

So the average probability of error

$$= \frac{1}{2}\left[P\left(\frac{s_{21}\text{ received}}{s_{11}\text{ transmitted}}\right) + P\left(\frac{s_{11}\text{ received}}{s_{21}\text{ transmitted}}\right)\right]$$

$$= \frac{1}{2}\left[Q\left(\sqrt{\frac{E}{N_0}}\right) + Q\left(\sqrt{\frac{E}{N_0}}\right)\right] = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

MCQ 1.28 A transmission line with a characteristic impedance of $100\,\Omega$ is used to match a $50\,\Omega$ section to a $200\,\Omega$ section. If the matching is to be done both at 429 MHz and 1 GHz, the length of the transmission line can be approximately

(A) 82.5 cm (b) 1.05 m

(C) 1.58 cm (D) 1.75 m

SOL 1.28 Option (C) is correct.

Since

$$Z_0 = \sqrt{Z_1 Z_2}$$

$$100 = \sqrt{50 \times 200}$$

This is quarter wave matching. The length would be odd multiple of $\lambda/4$.

$$l = (2m+1)\frac{\lambda}{4}$$

$f_1 = 429$ MHz, $\quad l_1 = \dfrac{c}{f_1 \times 4} = \dfrac{3 \times 10^8}{429 \times 10^6 \times 4} = 0.174$ m

$f_2 = 1$ GHz, $\quad l_2 = \dfrac{c}{f_2 \times 4} = \dfrac{3 \times 10^8}{1 \times 10^9 \times 4} = 0.075$ m

Only option (C) is odd multiple of both l_1 and l_2.

$$(2m+1) = \frac{1.58}{l_1} = 9$$

$$(2m+1) = \frac{1.58}{l_2} \simeq 21$$

MCQ 1.29 The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$. The system is

(A) time-invariant and stable
(B) stable and not time-invariant
(C) time-invariant and not stable
(D) not time-invariant and not stable

SOL 1.29 Option (D) is correct.

$$y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$

Time invariance :

Let, $x(t) = \delta(t)$

$$y(t) = \int_{-\infty}^{t} \delta(t)\cos(3\tau)\,d\tau$$
$$= u(t)\cos(0)$$
$$= u(t)$$

For a delayed input $(t-t_0)$ output is

$$y(t,t_0) = \int_{-\infty}^{t} \delta(t-t_0)\cos(3\tau)\,d\tau$$
$$= u(t)\cos(3t_0)$$

Delayed output

$$y(t-t_0) = u(t-t_0)$$
$$y(t,t_0) \neq y(t-t_0)$$

System is not time invariant.

Stability :

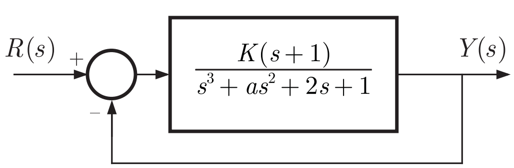
Consider a bounded input $x(t) = \cos 3t$

$$y(t) = \int_{-\infty}^{t} \cos^2 3t = \int_{-\infty}^{t} \frac{1-\cos 6t}{2}$$
$$= \frac{1}{2}\int_{-\infty}^{t} 1\,dt - \frac{1}{2}\int_{-\infty}^{t} \cos 6t\,dt$$

As $t \to \infty$, $y(t) \to \infty$ (unbounded)

System is not stable.

MCQ 1.30 The feedback system shown below oscillates at 2 rad/s when



(A) $K = 2$ and $a = 0.75$
(B) $K = 3$ and $a = 0.75$
(C) $K = 4$ and $a = 0.5$
(D) $K = 2$ and $a = 0.5$

SOL 1.30 Option (A) is correct.

$$Y(s) = \frac{K(s+1)}{s^3 + as^2 + 2s + 1}[R(s) - Y(s)]$$

$$Y(s)\left[1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1}\right] = \frac{K(s+1)}{s^3 + as^2 + 2s + 1}R(s)$$

$$Y(s)\,[s^3 + as^2 + s(2+k) + (1+k)] = K(s+1)\,R(s)$$

Transfer Function

$$H(s) = \frac{Y(s)}{R(s)} = \frac{K(s+1)}{s^3 + as^2 + s(2+k) + (1+k)}$$

Routh Table :

s^3	1	$2+K$
s^2	a	$1+K$
s^1	$\frac{a(2+K)-(1+K)}{a}$	0

For oscillation,

$$\frac{a(2+K)-(1+K)}{a} = 0$$

$$a = \frac{K+1}{K+2}$$

Auxiliary equation

$$A(s) = as^2 + (k+1) = 0$$

$$s^2 = -\frac{k+1}{a}$$

$$s^2 = \frac{-k+1}{(k+1)}(k+2)$$

$$s^2 = -(k+2)$$

$$s = j\sqrt{k+2}$$

$$j\omega = j\sqrt{k+2}$$

$$\omega = \sqrt{k+2} = 2 \qquad \text{(Oscillation frequency)}$$

$$k = 2$$

and $$a = \frac{2+1}{2+2} = \frac{3}{4} = 0.75$$

MCQ 1.31 The Fourier transform of a signal $h(t)$ is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of $h(0)$ is

(A) 1/4 (B) 1/2

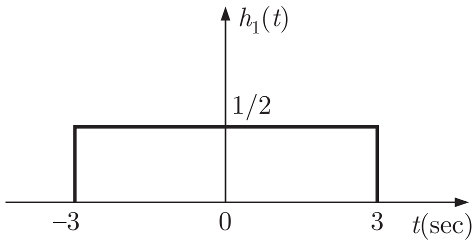
(C) 1 (D) 2

SOL 1.31 Option (C) is correct.

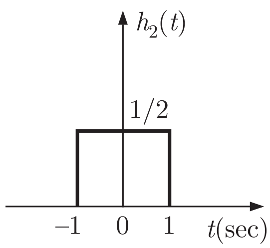
$$H(j\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega}$$

$$= \frac{\sin 3\omega}{\omega} + \frac{\sin \omega}{\omega}$$

We know that inverse Fourier transform of $\sin c$ function is a rectangular function.



$$\xleftrightarrow{\mathcal{F}} \quad \frac{\sin 3\omega}{\omega}$$



$$\xleftrightarrow{\mathcal{F}} \quad \frac{\sin \omega}{\omega}$$

So, inverse Fourier transform of $H(j\omega)$

$$h(t) = h_1(t) + h_2(t)$$
$$h(0) = h_1(0) + h_2(0)$$
$$= \frac{1}{2} + \frac{1}{2} = 1$$

MCQ 1.32 The state variable description of an LTI system is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

(A) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

(B) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(C) $a_1 = 0, a_3 \neq 0, a_3 = 0$

(D) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

SOL 1.32 Option (D) is correct.

General form of state equations are given as

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u$$
$$\dot{y} = \boldsymbol{C}x + \boldsymbol{D}u$$

For the given problem

$$\boldsymbol{A} = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{AB} = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ a_2 \\ 0 \end{bmatrix}$$

$$\boldsymbol{A}^2\boldsymbol{B} = \begin{bmatrix} 0 & 0 & a_1 a_2 \\ a_2 a_3 & 0 & 0 \\ 0 & a_3 a_1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 a_2 \\ 0 \\ 0 \end{bmatrix}$$

For controllability it is necessary that following matrix has a tank of $n = 3$.

$$\boldsymbol{U} = [\boldsymbol{B} : \boldsymbol{AB} : \boldsymbol{A}^2\boldsymbol{B}]$$

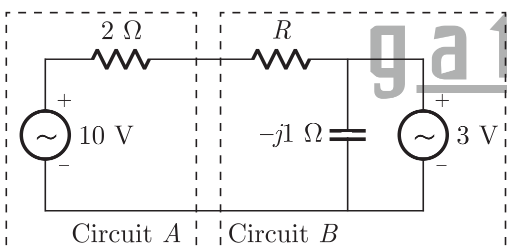
$$= \begin{bmatrix} 0 & 0 & a_1 a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

So, $a_2 \neq 0$

$$a_1 a_2 \neq 0 \Rightarrow a_1 \neq 0$$

a_3 may be zero or not.

MCQ 1.33 Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is

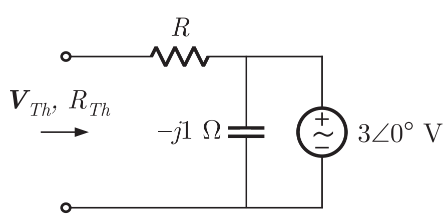


(A) $0.8\,\Omega$ (B) $1.4\,\Omega$

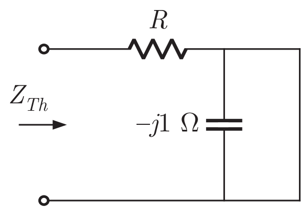
(C) $2\,\Omega$ (D) $2.8\,\Omega$

SOL 1.33 Option (A) is correct.

We obtain Thevenin equivalent of circuit B.



Thevenin Impedance :

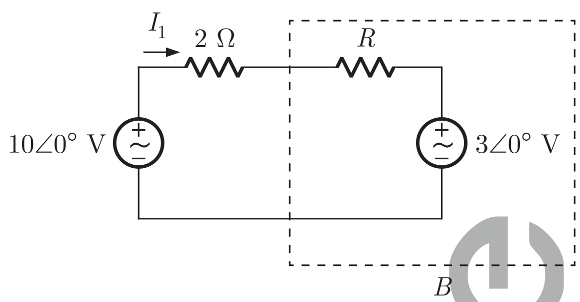


$$Z_{Th} = R$$

Thevenin Voltage :

$$\boldsymbol{V}_{Th} = 3\angle 0^\circ \text{ V}$$

Now, circuit becomes as



Current in the circuit, $I_1 = \dfrac{10-3}{2+R}$

Power transfer from circuit A to B

$$P = (I_1^2)^2 R + 3I_1$$

$$P = \left[\frac{10-3}{2+R}\right]^2 R + 3\left[\frac{10-3}{2+R}\right]$$

$$P = \frac{49R}{(2+R)^2} + \frac{21}{(2+R)}$$

$$P = \frac{49R + 21(2+R)}{(2+R)^2}$$

$$P = \frac{42 + 70R}{(2+R)^2}$$

$$\frac{dP}{dR} = \frac{(2+R)^2 70 - (42+70R)2(2+R)}{(2+R)^4} = 0$$

$$(2+R)[(2+R)70 - (42+70R)2] = 0$$

$$140 + 70R - 84 - 140R = 0$$

$$56 = 70R$$

$$R = 0.8\,\Omega$$

MCQ 1.34 Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t) \text{ with } y(t)\Big|_{t=0^-} = -2 \text{ and } \frac{dy}{dt}\Big|_{t=0^-} = 0$$

The numerical value of $\left.\frac{dy}{dt}\right|_{t=0^+}$ is

(A) -2 (B) -1

(C) 0 (D) 1

SOL 1.34 Option (D) is correct.

$$\frac{d^2y(t)}{dt^2}+\frac{2dy(t)}{dt}+y(t)=\delta(t)$$

By taking Laplace transform with initial conditions

$$\left[s^2Y(s)-sy(0)-\left.\frac{dy}{dt}\right|_{t=0}\right]+2[sy(s)-y(0)]+Y(s)=1$$

$$\Rightarrow \qquad [s^2Y(s)+2s-0]+2[sY(s)+2]+Y(s)=1$$

$$Y(s)[s^2+2s+1]=1-2s-4$$

$$Y(s)=\frac{-2s-3}{s^2+2s+1}$$

We know that,

If, $$y(t)\overset{\mathcal{L}}{\longleftrightarrow}Y(s)$$

then, $$\frac{dy(t)}{dt}\overset{\mathcal{L}}{\longleftrightarrow}sY(s)-y(0)$$

So, $$sY(s)-y(0)=\frac{(-2s-3)s}{(s^2+2s+1)}+2$$

$$=\frac{-2s^2-3s+2s^2+4s+2}{(s^2+2s+1)}$$

$$sY(s)-y(0)=\frac{s+2}{(s+1)^2}=\frac{s+1}{(s+1)^2}+\frac{1}{(s+1)^2}$$

$$=\frac{1}{s+1}+\frac{1}{(s+1)^2}$$

By taking inverse Laplace transform

$$\frac{dy(t)}{dt}=e^{-t}u(t)+te^{-t}u(t)$$

At $t=0^+$, $$\left.\frac{dy}{dt}\right|_{t=0^+}=e^0+0=1$$

MCQ 1.35 The direction of vector $\boldsymbol{A}$ is radially outward from the origin, with $|\boldsymbol{A}|=kr^n$. where $r^2=x^2+y^2+z^2$ and k is a constant. The value of n for which $\nabla\cdot\boldsymbol{A}=0$ is

(A) -2 (B) 2

(C) 1 (D) 0

SOL 1.35 Option (A) is correct.

Divergence of $\boldsymbol{A}$ in spherical coordinates is given as

$$\nabla\cdot\boldsymbol{A}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(kr^{n+2})$$

$$= \frac{k}{r^2}(n+2)r^{n+1}$$

$$= k(n+2)r^{n-1} = 0 \quad \text{(given)}$$

$$n+2 = 0$$

$$n = -2$$

MCQ 1.36 A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

(A) 1/3 (B) 1/2

(C) 2/3 (D) 3/4

SOL 1.36 Option (C) is correct.

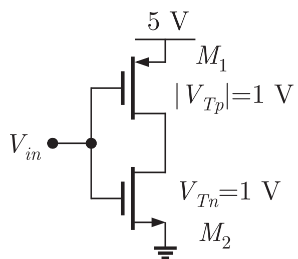
Probability of appearing a head is 1/2. If the number of required tosses is odd, we have following sequence of events.

$$H, TTH, TTTTH, \ldots\ldots\ldots$$

Probability $$P = \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots..$$

$$P = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$$

MCQ 1.37 In the CMOS circuit shown, electron and hole mobilities are equal, and M_1 and M_2 are equally sized. The device M_1 is in the linear region if

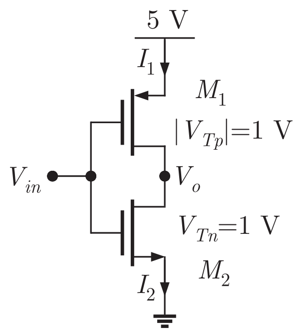


(A) $V_{in} < 1.875$ V (B) $1.875\text{ V} < V_{in} < 3.125$ V

(C) $V_{in} > 3.125$ V (D) $0 < V_{in} < 5$ V

SOL 1.37 Option (A) is correct.

Given the circuit as below :



Since all the parameters of PMOS and NMOS are equal.

So, $$\mu_n = \mu_p$$

$$C_{OX}\left(\frac{W}{L}\right)_{M_1} = C_{OX}\left(\frac{W}{L}\right)_{M_2} = C_{OX}\left(\frac{W}{L}\right)$$

Given that M_1 is in linear region. So, we assume that M_2 is either in cutoff or saturation.

Case 1 : M_2 is in cut off

So, $$I_2 = I_1 = 0$$

Where I_1 is drain current in M_1 and I_2 is drain current in M_2.

Since, $$I_1 = \frac{\mu_p C_{OX}}{2}\left(\frac{W}{L}\right)\left[2V_{SD}(V_{SG} - V_{Tp}) - V_{SD}^2\right]$$

$$\Rightarrow \quad 0 = \frac{\mu_p C_{OX}}{2}\left(\frac{W}{L}\right)[2V_{SD}(V_{SG} - V_{Tp}) - V_{SD}^2]$$

Solving it we get,

$$2(V_{SG} - V_{Tp}) = V_{SD}$$

$$\Rightarrow \quad 2(5 - V_{in} - 1) = 5 - V_D$$

$$\Rightarrow \quad V_{in} = \frac{V_D + 3}{2}$$

For $$I_1 = 0,\ V_D = 5\text{ V}$$

So, $$V_{in} = \frac{5+3}{2} = 4\text{ V}$$

So for the NMOS

$$V_{GS} = V_{in} - 0$$
$$= 4 - 0 = 4\text{ V and } V_{GS} > V_{Tn}$$

So it can't be in cutoff region.

Case 2 : M_2 must be in saturation region.

So, $$I_1 = I_2$$

$$\frac{\mu_p C_{OX}}{2}\frac{W}{L}\left[2(V_{SG} - V_{Tp})V_{SD} - V_{SD}^2\right] = \frac{\mu_n C_{OX}}{2}\frac{W}{L}(V_{GS} - V_{Tn})^2$$

$$\Rightarrow \quad 2(V_{SG} - V_{Tp})V_{SD} - V_{SD}^2 = (V_{GS} - V_{Tn})^2$$

$$\Rightarrow \quad 2(5 - V_{in} - 1)(5 - V_D) - (5 - V_D)^2 = (V_{in} - 0 - 1)^2$$

$$\Rightarrow \quad 2(4 - V_{in})(5 - V_D) - (5 - V_D)^2 = (V_{in} - 1)^2$$

Substituting $V_D = V_{DS} = V_{GS} - V_{Tn}$ and for N-MOS $\Rightarrow V_D = V_{in} - 1$

$$\Rightarrow \quad 2(4 - V_{in})(6 - V_{in}) - (6 - V_{in})^2 = (V_{in} - 1)^2$$

$$\Rightarrow \quad 48 - 36 - 8V_{in} = -2V_{in} + 1$$

$$\Rightarrow \quad 6V_{in} = 11$$

$$\Rightarrow \quad V_{in} = \frac{11}{6} = 1.833\text{ V}$$

So for M_2 to be in saturation $V_{in} < 1.833$ V or $V_{in} < 1.875$ V

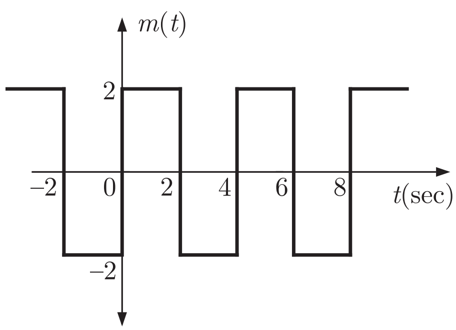
MCQ 1.38 A binary symmetric channel (BSC) has a transition probability of 1/8. If the binary symbol X is such that $P(X = 0) = 9/10$, then the probability of error for an optimum receiver will be

(A) 7/80 (B) 63/80

(C) 9/10 (D) 1/10

SOL 1.38

MCQ 1.39 The signal $m(t)$ as shown is applied to both a phase modulator (with k_p as the phase constant) and a frequency modulator (with k_f as the frequency constant) having the same carrier frequency.



The ratio k_p/k_f (in rad/Hz) for the same maximum phase deviation is

(A) 8π (B) 4π

(C) 2π (D) π

SOL 1.39 Option (B) is correct.

General equation of FM and PM waves are given by

$$\phi_{FM}(t) = A_c \cos\left[\omega_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

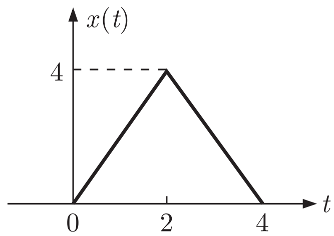
$$\phi_{PM}(t) = A_c \cos[\omega_c t + k_p m(t)]$$

For same maximum phase deviation.

$$k_p[m(t)]_{\max} = 2\pi k_f \left[\int_0^t m(\tau)\, d\tau\right]_{\max}$$

$$k_p \times 2 = 2\pi k_f [x(t)]_{\max}$$

where, $$x(t) = \int_0^t m(\tau)\, d\tau$$



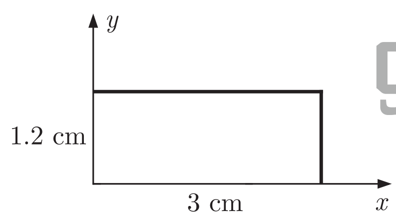
$$[x(t)]_{\max} = 4$$

So, $$k_p \times 2 = 2\pi k_f \times 4$$

$$\frac{k_p}{k_f} = 4\pi$$

MCQ 1.40 The magnetic field among the propagation direction inside a rectangular waveguide with the cross-section shown in the figure is

$$H_z = 3\cos(2.094 \times 10^2 x)\cos(2.618 \times 10^2 y)\cos(6.283 \times 10^{10} t - \beta z)$$



The phase velocity v_p of the wave inside the waveguide satisfies

(A) $v_p > c$

(B) $v_p = c$

(C) $0 < v_p < c$

(D) $v_p = 0$

SOL 1.40 Option (D) is correct.

$$H_z = 3\cos(2.094 \times 10^2 x)\cos(2.618 \times 10^2 y)\cos(6.283 \times 10^{10} t - \beta z)$$

$$\beta_x = 2.094 \times 10^2$$

$$\beta_y = 2.618 \times 10^2$$

$$\omega = 6.283 \times 10^{10} \text{ rad/s}$$

For the wave propagation,

$$\beta = \sqrt{\frac{\omega^2}{c^2} - (\beta_x^2 + \beta_y^2)}$$

Substituting above values,

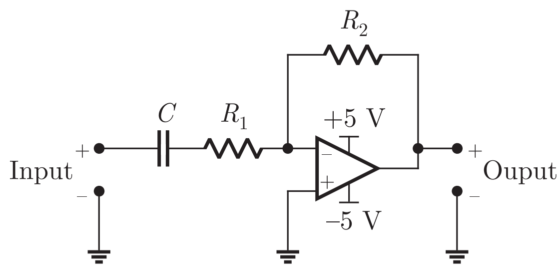
$$\beta = \sqrt{\left(\frac{6.283 \times 10^{10}}{3 \times 10^8}\right)^2 - (2.094^2 + 2.618^2) \times 10^4}$$

$$\simeq j261$$

β is imaginary so mode of operation is non-propagating.

$$v_p = 0$$

MCQ 1.41 The circuit shown is a



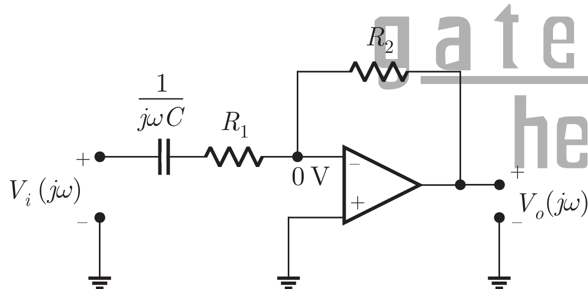
(A) low pass filter with $f_{3dB} = \frac{1}{(R_1 + R_2)C}$ rad/s

(B) high pass filter with $f_{3dB} = \frac{1}{R_1 C}$ rad/s

(C) low pass filter with $f_{3dB} = \frac{1}{R_1 C}$ rad/s

(D) high pass filter with $f_{3dB} = \frac{1}{(R_1 + R_2)C}$ rad/s

SOL 1.41 Option (B) is correct.
First we obtain the transfer function.



$$\frac{0 - V_i(j\omega)}{\frac{1}{j\omega C} + R_1} + \frac{0 - V_o(j\omega)}{R_2} = 0$$

$$\frac{V_o(j\omega)}{R_2} = \frac{-V_i(j\omega)}{\frac{1}{j\omega C} + R_1}$$

$$V_o(j\omega) = -\frac{V_i(j\omega)R_2}{R_1 - j\frac{1}{\omega C}}$$

At $\omega \to 0$ (Low frequencies)

$$\frac{1}{\omega C} \to \infty, \text{ so } V_o = 0$$

At $\omega \to \infty$ (higher frequencies)

$$\frac{1}{\omega C} \to 0, \text{ so } V_o(j\omega) = -\frac{R_2}{R_1}V_i(j\omega)$$

The filter passes high frequencies so it is a high pass filter.

$$H(j\omega) = \frac{V_o}{V_i} = \frac{-R_2}{R_1 - j\frac{1}{\omega C}}$$

$$|H(\infty)| = \left|\frac{-R_2}{R_1}\right| = \frac{R_2}{R_1}$$

At 3 dB frequency, gain will be $\sqrt{2}$ times of maximum gain $[H(\infty)]$

$$|H(j\omega_0)| = \frac{1}{\sqrt{2}}|H(\infty)|$$

So,

$$\frac{R_2}{\sqrt{R_1^2 + \frac{1}{\omega_0^2 C^2}}} = \frac{1}{\sqrt{2}}\left(\frac{R_2}{R_1}\right)$$

$$2R_1^2 = R_1^2 + \frac{1}{\omega_0^2 C^2}$$

$$R_1^2 = \frac{1}{\omega^2 C^2}$$

$$\omega_0 = \frac{1}{R_1 C}$$

MCQ 1.42 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals

(A) 0 (B) 1/2

(C) 1 (D) 3/2

SOL 1.42 Option (A) id correct.

Convolution sum is defined as

$$y[n] = h[n] * g[n] = \sum_{k=-\infty}^{\infty} h[n]\, g[n-k]$$

For causal sequence,

$$y[n] = \sum_{k=0}^{\infty} h[n]\, g[n-k]$$

$$y[n] = h[n]\, g[n] + h[n]\, g[n-1] + h[n]\, g[n-2] + ...$$

For $n = 0$,

$$y[0] = h[0]\, g[0] + h[1]\, g[-1] + ...$$

$$y[0] = h[0]\, g[0] \qquad g[-1] = g[-2] = ...0$$

$$y[0] = h[0]\, g[0] \qquad ...(i)$$

For $n = 1$,

$$y[1] = h[1]\, g[1] + h[1]\, g[0] + h[1]\, g[-1] + ...$$

$$y[1] = h[1]\, g[1] + h[1]\, g[0]$$

$$\frac{1}{2} = \frac{1}{2} g[1] + \frac{1}{2} g[0] \qquad h[1] = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

$$1 = g[1] + g[0]$$

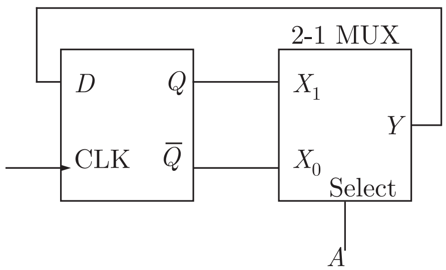
$$g[1] = 1 - g[0]$$

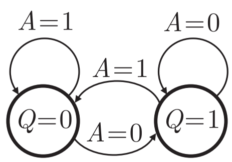
From equation (i),

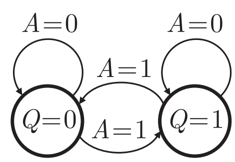
$$g[0] = \frac{y[0]}{h[0]} = \frac{1}{1} = 1$$

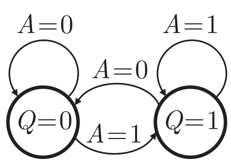
So, $$g[1] = 1 - 1 = 0$$

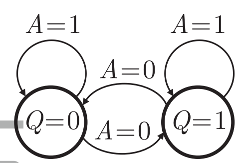
MCQ 1.43 The state transition diagram for the logic circuit shown is



(A)


(B)


(C)


(B)


SOL 1.43 Option (D) is correct.

Let Q_{n+1} is next state and Q_n is the present state. From the given below figure.

$$D = Y = \overline{A}X_0 + AX_1$$

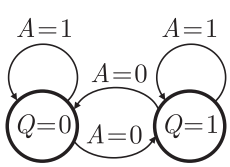
$$Q_{n+1} = D = \overline{A}X_0 + AX_1$$

$$Q_{n+1} = \overline{A}\,\overline{Q_n} + A\,Q_n \qquad X_0 = \overline{Q},\ X_1 = Q$$

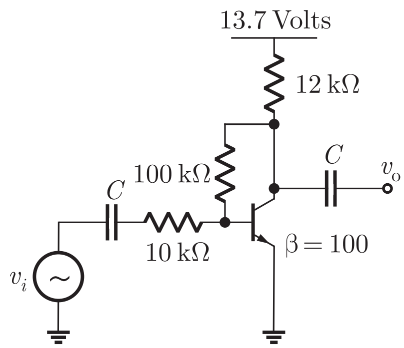
If $A = 0$, $\quad Q_{n+1} = \overline{Q_n}$ (toggle of previous state)

If $A = 1$, $\quad Q_{n+1} = Q_n$

So state diagram is



MCQ 1.44 The voltage gain A_v of the circuit shown below is

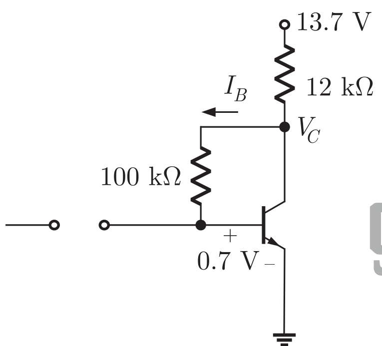


(A) $|A_v| \approx 200$ (B) $|A_v| \approx 100$

(C) $|A_v| \approx 20$ (D) $|A_v| \approx 10$

SOL 1.44 Option (D) is correct.

DC Analysis :



Using KVL in input loop,

$$V_C - 100 I_B - 0.7 = 0$$

$$V_C = 100 I_B + 0.7 \qquad \text{...(i)}$$

$$I_C \simeq I_E = \frac{13.7 - V_C}{12k} = (\beta + 1) I_B$$

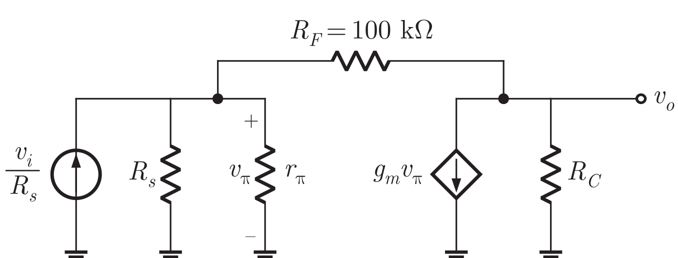
$$\frac{13.7 - V_C}{12 \times 10^3} = 100 I_B \qquad \text{...(ii)}$$

Solving equation (i) and (ii),

$$I_B = 0.01\,\text{mA}$$

Small Signal Analysis :

Transforming given input voltage source into equivalent current source.



This is a shunt-shunt feedback amplifier.
Given parameters,

$$r_\pi = \frac{V_T}{I_B} = \frac{25\,\text{mV}}{0.01\,\text{mA}} = 2.5\,\text{k}\Omega$$

$$g_m = \frac{\beta}{r_\pi} = \frac{100}{2.5 \times 1000} = 0.04\,\text{s}$$

Writing KCL at output node

$$\frac{v_0}{R_C} + g_m v_\pi + \frac{v_0 - v_\pi}{R_F} = 0$$

$$v_0\left[\frac{1}{R_C} + \frac{1}{R_F}\right] + v_\pi\left[g_m - \frac{1}{R_F}\right] = 0$$

Substituting $R_C = 12\,\text{k}\Omega,\ R_F = 100\,\text{k}\Omega,\ g_m = 0.04\,\text{s}$

$$v_0(9.33 \times 10^{-5}) + v_\pi(0.04) = 0$$

$$v_0 = -\,428.72\,V_\pi \qquad \text{...(i)}$$

Writing KCL at input node

$$\frac{v_i}{R_s} = \frac{v_\pi}{R_s} + \frac{v_\pi}{r_\pi} + \frac{v_\pi - v_o}{R_F}$$

$$\frac{v_i}{R_s} = v_\pi\left[\frac{1}{R_s} + \frac{1}{r_\pi} + \frac{1}{R_F}\right] - \frac{v_0}{R_F}$$

$$\frac{v_i}{R_s} = v_\pi(5.1 \times 10^{-4}) - \frac{v_0}{R_F}$$

Substituting V_π from equation (i)

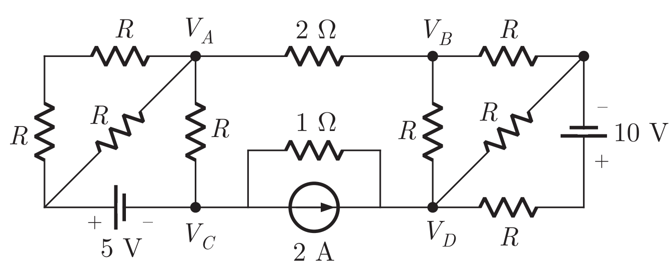
$$\frac{v_i}{R_s} = \frac{-5.1 \times 10^{-4}}{428.72}v_0 - \frac{v_0}{R_F}$$

$$\frac{v_i}{10 \times 10^3} = -\,1.16 \times 10^{-6}v_0 - 1 \times 10^{-5}v_0 \qquad R_s = 10\,\text{k}\Omega \text{ (source resistance)}$$

$$\frac{v_i}{10 \times 10^3} = -\,1.116 \times 10^{-5}$$

$$|A_v| = \left|\frac{v_0}{v_i}\right| = \frac{1}{10 \times 10^3 \times 1.116 \times 10^{-5}} \simeq 8.96$$

MCQ 1.45 If $V_A - V_B = 6$ V then $V_C - V_D$ is



(A) -5 V
(B) 2 V
(C) 3 V
(D) 6 V

SOL 1.45 Option (A) is correct.

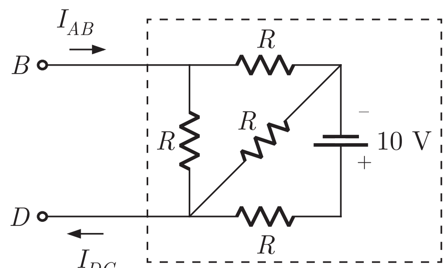
In the given circuit

$$V_A - V_B = 6\text{ V}$$

So current in the branch will be

$$I_{AB} = \frac{6}{2} = 3\text{ A}$$

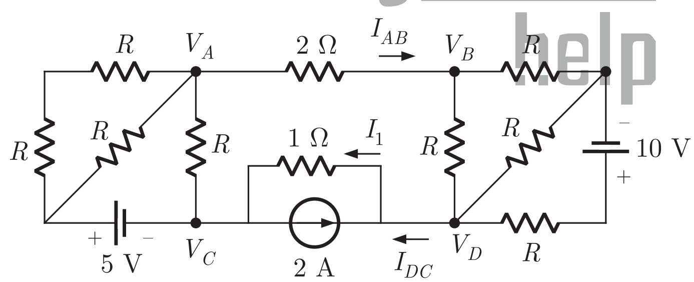
We can see, that the circuit is a one port circuit looking from terminal BD as shown below



One port network

For a one port network current entering one terminal, equals the current leaving the second terminal. Thus the outgoing current from A to B will be equal to the incoming current from D to C as shown

i.e. $$I_{DC} = I_{AB} = 3\text{ A}$$



The total current in the resistor $1\,\Omega$ will be

$$I_1 = 2 + I_{DC} \qquad \text{(By writing KCL at node } D\text{)}$$

$$= 2 + 3 = 5\text{ A}$$

So, $$V_{CD} = 1 \times (-I_1)$$

$$= -5\text{ V}$$

MCQ 1.46 The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval $[1,6]$ is

(A) 21 (B) 25

(C) 41 (D) 46

SOL 1.46 Option (B) is correct.

$$f(x) = x^3 - 9x^2 + 24x + 5$$

$$\frac{df(x)}{dx} = 3x^2 - 18x + 24 = 0$$

$$\Rightarrow \qquad \frac{df(x)}{dx} = x^2 - 6x + 8 = 0$$

$x = 4, \; x = 2$

$$\frac{d^2 f(x)}{dx^2} = 6x - 18$$

For $x = 2, \; \frac{d^2 f(x)}{dx^2} = 12 - 18 = -6 < 0$

So at $x = 2, \; f(x)$ will be maximum

$$\begin{aligned} f(x)\big|_{max} &= (2)^3 - 9(2)^2 + 24(2) + 5 \\ &= 8 - 36 + 48 + 5 \\ &= 25 \end{aligned}$$

MCQ 1.47 Given that

$$\boldsymbol{A} = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \text{and } \boldsymbol{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ the value of } A^3 \text{ is}$$

(A) $15\boldsymbol{A} + 12\boldsymbol{I}$

(B) $19\boldsymbol{A} + 30\boldsymbol{I}$

(C) $17\boldsymbol{A} + 15\boldsymbol{I}$

(D) $17\boldsymbol{A} + 21\boldsymbol{I}$

SOL 1.47 Option (B) is correct.

Characteristic equation.

$$|\boldsymbol{A} - \lambda \boldsymbol{I}| = 0$$

$$\begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$$

$$5\lambda + \lambda^2 + 6 = 0$$

$$\lambda^2 + 5\lambda + 6 = 0$$

Since characteristic equation satisfies its own matrix, so

$$\boldsymbol{A}^2 + 5\boldsymbol{A} + 6 = 0 \Rightarrow \boldsymbol{A}^2 = -5\boldsymbol{A} - 6\boldsymbol{I}$$

Multiplying with $\boldsymbol{A}$

$$\begin{aligned} \boldsymbol{A}^3 + 5\boldsymbol{A}^2 + 6\boldsymbol{A} &= 0 \\ \boldsymbol{A}^3 + 5(-5\boldsymbol{A} - 6\boldsymbol{I}) + 6\boldsymbol{A} &= 0 \\ \boldsymbol{A}^3 &= 19\boldsymbol{A} + 30\boldsymbol{I} \end{aligned}$$

Common Data Questions

Common Data for Questions 48 and 49 :

With 10 V dc connected at port A in the linear nonreciprocal two-port network shown below, the following were observed :

(i) $1\,\Omega$ connected at port B draws a current of 3 A

(ii) $2.5\,\Omega$ connected at port B draws a current of 2 A

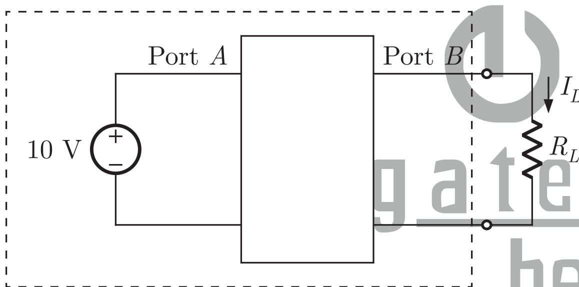
MCQ 1.48 With 10 V dc connected at port A, the current drawn by $7\,\Omega$ connected at port B is

(A) 3/7 A (B) 5/7 A

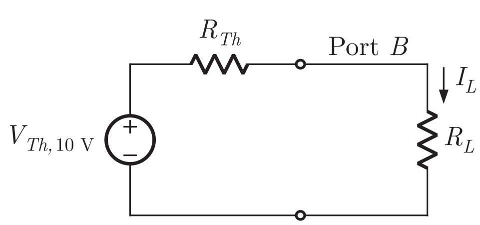
(C) 1 A (D) 9/7 A

SOL 1.48 Option (C) is correct.

When 10 V is connected at port A the network is



Now, we obtain Thevenin equivalent for the circuit seen at load terminal, let Thevenin voltage is $V_{Th,10\,\mathrm{V}}$ with 10 V applied at port A and Thevenin resistance is R_{Th}.



$$I_L = \frac{V_{Th,10\,\mathrm{V}}}{R_{Th} + R_L}$$

For $R_L = 1\,\Omega$, $I_L = 3$ A

$$3 = \frac{V_{Th,10\,\mathrm{V}}}{R_{Th} + 1} \qquad \text{...(i)}$$

For $R_L = 2.5\,\Omega$, $I_L = 2$ A

$$= \frac{V_{Th,10\,\mathrm{V}}}{R_{Th} + 2.5} \qquad \text{...(ii)}$$

Dividing above two

$$\frac{3}{2} = \frac{R_{Th} + 2.5}{R_{Th} + 1}$$

$$3R_{Th} + 3 = 2R_{Th} + 5$$

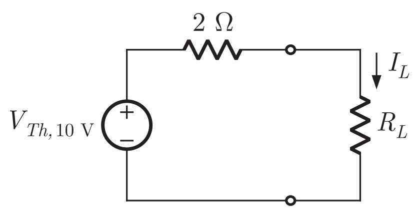
$$R_{Th} = 2\,\Omega$$

Substituting R_{Th} into equation (i)

$$V_{Th,10\,\mathrm{V}} = 3(2+1) = 9\ \mathrm{V}$$

Note that it is a non reciprocal two port network. Thevenin voltage seen at port B depends on the voltage connected at port A. Therefore we took subscript $V_{Th,10\,\mathrm{V}}$. This is Thevenin voltage only when 10 V source is connected at input port A. If the voltage connected to port A is different, then Thevenin voltage will be different. However, Thevenin's resistance remains same.

Now, the circuit is



For $R_L = 7\,\Omega$

$$I_L = \frac{V_{Th,10\,\mathrm{V}}}{2 + R_L} = \frac{9}{2+7} = 1\ \mathrm{A}$$

MCQ 1.49 For the same network, with 6 V dc connected at port A, $1\,\Omega$ connected at port B draws $7/3$ A. If 8 V dc is connected to port A, the open circuit voltage at port B is

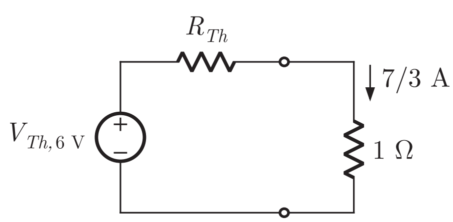
(A) 6 V (B) 7 V

(C) 8 V (D) 9 V

SOL 1.49 Option (B) is correct.

Now, when 6 V connected at port A let Thevenin voltage seen at port B is $V_{Th,6\,\mathrm{V}}$.

Here $R_L = 1\,\Omega$ and $I_L = \frac{7}{3}$ A



$$V_{Th,6\,\mathrm{V}} = R_{Th} \times \frac{7}{3} + 1 \times \frac{7}{3}$$

$$= 2 \times \frac{7}{3} + \frac{7}{3} = 7\ \mathrm{V}$$

This is a linear network, so V_{Th} at port B can be written as

$$V_{Th} = V_1\alpha + \beta$$

where V_1 is the input applied at port A.

We have $V_1 = 10\text{ V}$, $V_{Th,10\text{ V}} = 9\text{ V}$

$\therefore$ $$9 = 10\alpha + \beta \quad \text{...(i)}$$

When $V_1 = 6\text{ V}$, $V_{Th,6\text{ V}} = 9\text{ V}$

$\therefore$ $$7 = 6\alpha + \beta \quad \text{...(ii)}$$

Solving (i) and (ii)

$$\alpha = 0.5,\ \beta = 4$$

Thus, with any voltage V_1 applied at port A, Thevenin voltage or open circuit voltage at port B will be

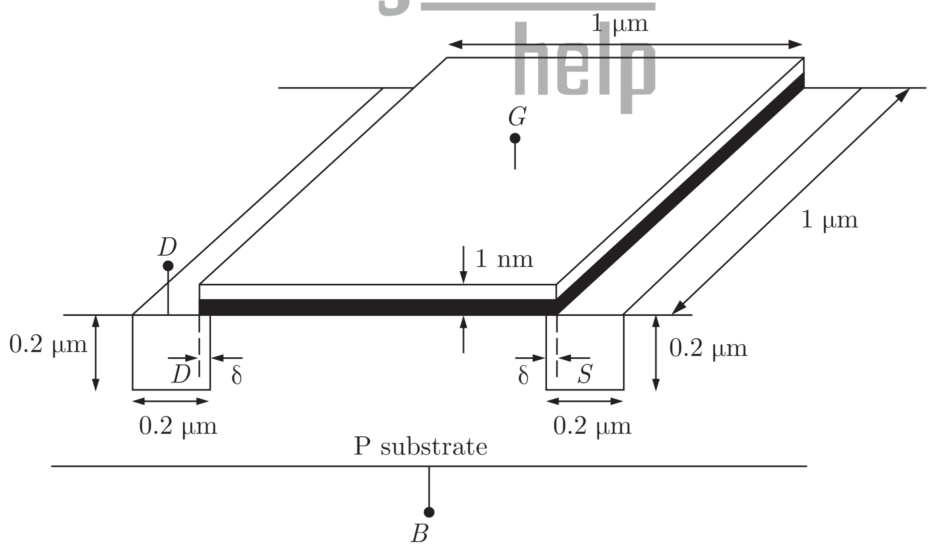
So, $$V_{Th,V_1} = 0.5V_1 + 4$$

For $$V_1 = 8\text{ V}$$

$$V_{Th,8\text{ V}} = 0.5 \times 8 + 4 = 8 = V_{oc} \quad \text{(open circuit voltage)}$$

Common Data for Question 50 and 51 :

In the three dimensional view of a silicon n-channel MOS transistor shown below, $\delta = 20\text{ nm}$. The transistor is of width $1\,\mu\text{m}$. The depletion width formed at every p-n junction is 10 nm. The relative permittivity of Si and SiO_2, respectively, are 11.7 and 3.9, and $\varepsilon_0 = 8.9 \times 10^{-12}$ F/m.



MCQ 1.50 The gate source overlap capacitance is approximately

(A) 0.7 fF (B) 0.7 pF

(C) 0.35 fF (D) 0.24 pF

SOL 1.50 Option (B) is correct.

Gate source overlap capacitance.

$$C_o = \frac{\delta W \varepsilon_{ox}\varepsilon_0}{t_{ox}} \quad (\text{medium } Sio_2)$$

$$= \frac{20 \times 10^{-9} \times 1 \times 10^{-6} \times 3.9 \times 8.9 \times 10^{-12}}{1 \times 10^{-9}}$$

$$= 0.69 \times 10^{-15}\,\text{F}$$

MCQ 1.51 The source-body junction capacitance is approximately

(A) 2 fF (B) 7 fF

(C) 2 pF (D) 7 pF

SOL 1.51 Option (B) is correct.

Source body junction capacitance.

$$C_s = \frac{A\varepsilon_r\varepsilon_0}{d}$$

$$A = (0.2\,\mu\text{m} + 0.2\,\mu\text{m} + 0.2\,\mu\text{m}) \times 1\,\mu\text{m} + 2(0.2\,\mu\text{m} \times 0.2\,\mu\text{m})$$

$$= 0.68\,\mu\text{m}^2$$

$$d = 10\,\text{nm} \text{ (depletion width of all junction)}$$

$$C_s = \frac{0.68 \times 10^{-12} \times 11.7 \times 8.9 \times 10^{-12}}{10 \times 10^{-9}}$$

$$= 7 \times 10^{-15}\,\text{F}$$

Linked Answer Questions

Statement for Linked Answer Question 52 and 53 :

An infinitely long uniform solid wire of radius a carries a uniform dc current of density J

MCQ 1.52 The magnetic field at a distance r from the center of the wire is proportional to

(A) r for $r < a$ and $1/r^2$ for $r > a$ (B) 0 for $r < a$ and $1/r$ for $r > a$

(C) r for $r < a$ and $1/r$ for $r > a$ (D) 0 for $r < a$ and $1/r^2$ for $r > a$

SOL 1.52 Option (C) is correct.

For $r > a$,

$$I_{enclosed} = (\pi a^2)\,J$$

$$\oint H \cdot dl = I_{enclosed}$$

$$H \times 2\pi r = (\pi a^2)\,J$$

$$H = \frac{I_o}{2\pi r} \qquad I_o = (\pi a^2)\,J$$

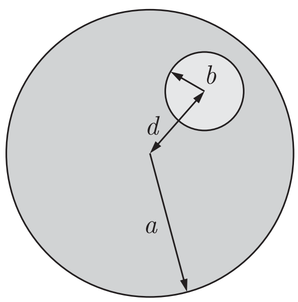
$$H \propto \frac{1}{r}, \quad \text{for } r > a$$

For $r < a$,

$$I_{enclosed} = \frac{J(\pi r^2)}{\pi a^2} = \frac{Jr^2}{a^2}$$

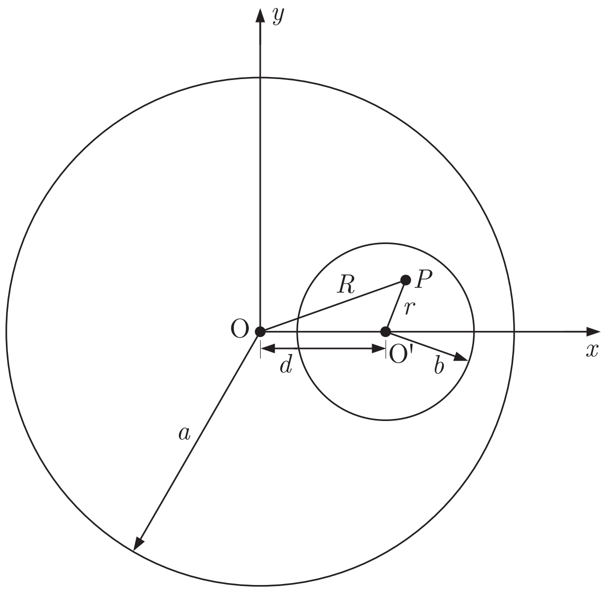
So,
$$\oint H \cdot dl = I_{enclosed}$$
$$H \times 2\pi r = \frac{Jr^2}{a^2}$$
$$H = \frac{Jr}{2\pi a^2}$$
$$H \propto r, \quad \text{for } r < a$$

MCQ 1.53 A hole of radius $b(b < a)$ is now drilled along the length of the wire at a distance d from the center of the wire as shown below.



The magnetic field inside the hole is
(A) uniform and depends only on d
(B) uniform and depends only on b
(C) uniform and depends on both b and d
(D) non uniform

SOL 1.53 Option (B) is correct.
Assuming the cross section of the wire on x-y plane as shown in figure.



Since, the hole is drilled along the length of wire. So, it can be assumed that the drilled portion carriers current density of $-J$.

Now, for the wire without hole, magnetic field intensity at point P will be given as

$$H_{\phi1}(2\pi R) = J(\pi R^2)$$

$$H_{\phi1}(2\pi R) = \frac{JR}{2}$$

Since, point o is at origin. So, in vector form

$$\boldsymbol{H}_1 = \frac{J}{2}(x\boldsymbol{a}_x + y\boldsymbol{a}_y)$$

Again only due to the hole magnetic field intensity will be given as.

$$(H_{\phi2})(2\pi r) = -J(\pi r^2)$$

$$H_{\phi2} = \frac{-Jr}{2}$$

Again, if we take O' at origin then in vector form

$$\boldsymbol{H}_2 = \frac{-J}{2}(x'\boldsymbol{a}_x + y'\boldsymbol{a}_y)$$

where x' and y' denotes point 'P' in the new co-ordinate system.

Now the relation between two co-ordinate system will be.

$$x = x' + d$$

$$y = y'$$

So, $$\boldsymbol{H}_2 = \frac{-J}{2}[(x-d)\boldsymbol{a}_x + y\boldsymbol{a}_y]$$

So, total magnetic field intensity $= \boldsymbol{H}_1 + \boldsymbol{H}_2 = \frac{J}{2}d\boldsymbol{a}_x$

So, magnetic field inside the hole will depend only on 'd'.

Statement for Linked Answer Question 54 and 55 :

The transfer function of a compensator is given as

$$G_c(s) = \frac{s+a}{s+b}$$

MCQ 1.54 $G_c(s)$ is a lead compensator if

(A) $a = 1, b = 2$ (B) $a = 3, b = 2$

(C) $a = -3, b = -1$ (D) $a = 3, b = 1$

SOL 1.54 Option (A) is correct.

$$G_C(s) = \frac{s+a}{s+b} = \frac{j\omega + a}{j\omega + b}$$

Phase lead angle

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

$$\phi = \tan^{-1}\left(\frac{\frac{\omega}{a} - \frac{\omega}{b}}{1 + \frac{\omega^2}{ab}}\right)$$

$$= \tan^{-1}\left(\frac{\omega(b-a)}{ab + \omega^2}\right)$$

For phase-lead compensation $\phi > 0$

$$b - a > 0$$

$$b > a$$

Note: For phase lead compensator zero is nearer to the origin as compared to pole, so option (C) can not be true.

MCQ 1.55 The phase of the above lead compensator is maximum at

(A) $\sqrt{2}$ rad/s (B) $\sqrt{3}$ rad/s

(C) $\sqrt{6}$ rad/s (D) $1/\sqrt{3}$ rad/s

SOL 1.55 Option (A) is correct.

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

$$\frac{d\phi}{d\omega} = \frac{1/a}{1 + \left(\frac{\omega}{a}\right)^2} - \frac{1/b}{1 + \left(\frac{\omega}{b}\right)^2} = 0$$

$$\frac{1}{a} + \frac{\omega^2}{ab^2} = \frac{1}{b} + \frac{1}{b}\frac{\omega^2}{a^2}$$

$$\frac{1}{a} - \frac{1}{b} = \frac{\omega^2}{ab}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\omega = \sqrt{ab}$$

$$= \sqrt{1 \times 2} = \sqrt{2} \text{ rad/sec}$$

General Aptitude (GA) Question (Compulsory)

Q. 56 - Q. 60 carry one mark each.

MCQ 1.56 If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321}$

(A) 2.23 (B) 4.33

(C) 11.37 (D) 27.64

SOL 1.56 Option (D) is correct option.

Let $1.001 = x$

So in given data :

$$x^{1259} = 3.52$$
$$x^{2062} = 7.85$$

Again
$$\begin{aligned} x^{3321} &= x^{1259+2062} \\ &= x^{1259}x^{2062} \\ &= 3.52 \times 7.85 \\ &= 27.64 \end{aligned}$$

MCQ 1.57 Choose the most appropriate alternate from the options given below to complete the following sentence :

If the tired soldier wanted to lie down, he..................the mattress out on the balcony.

(A) should take (B) shall take

(C) should have taken (D) will have taken

SOL 1.57 Option (C) is correct.

MCQ 1.58 Choose the most appropriate word from the options given below to complete the following sentence :

Give the seriousness of the situation that he had to face, his........was impressive.

(A) beggary (B) nomenclature

(C) jealousy (D) nonchalance

SOL 1.58 Option (D) is correct.

MCQ 1.59 Which one of the following options is the closest in meaning to the word given below ?

Latitude

(A) Eligibility (B) Freedom

(C) Coercion (D) Meticulousness

SOL 1.59 Option (B) is correct.

MCQ 1.60 One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is **INCORRECT** ?

I requested that he should be given the driving test today instead of tomorrow.

(A) requested that (B) should be given

(C) the driving test (D) instead of tomorrow

SOL 1.60 Option (B) is correct.

Q. 61 - Q. 65 carry two marks each.

MCQ 1.61 **One of the legacies of the Roman legions was discipline. In the legious, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**

Which one of the following statements best sums up the meaning of the above passage ?

(A) Through regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.

(B) The legions were treated inhumanly as if the men were animals

(C) Disciplines was the armies inheritance from their seniors

(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

SOL 1.61 Option (A) is correct.

MCQ 1.62 Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money values of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

(A) 5 (B) 6

(C) 9 (D) 10

SOL 1.62 Option (A) is correct.

Let no. of notes of Rs.20 be x and no. of notes of Rs. 10 be y.

Then from the given data.

$$x + y = 14$$

$$20x + 10y = 230$$

Solving the above two equations we get

$$x = 9,\ y = 5$$

So, the no. of notes of Rs. 10 is 5.

MCQ 1.63 There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is

(A) 2 (B) 3

(C) 4 (D) 8

SOL 1.63 Option (A) is correct.

We will categorize the 8 bags in three groups as :
(i) $A_1A_2A_3$, (ii) $B_1B_2B_3$, (iii) C_1C_2
Weighting will be done as bellow :
1st weighting $\rightarrow$ $A_1A_2A_3$ will be on one side of balance and $B_1B_2B_3$ on the other. It may have three results as described in the following cases.
Case 1 : $A_1A_2A_3 = B_1B_2B_3$
This results out that either C_1 or C_2 will heavier for which we will have to perform weighting again.
2nd weighting $\rightarrow$ C_1 is kept on the one side and C_2 on the other.
if $C_1 > C_2$ then C_1 is heavier.
$C_1 < C_2$ then C_2 is heavier.
Case 2 : $A_1A_2A_3 > B_1B_2B_3$
it means one of the $A_1A_2A_3$ will be heavier So we will perform next weighting as:
2nd weighting $\rightarrow$ A_1 is kept on one side of the balance and A_2 on the other.
if $A_1 = A_2$ it means A_3 will be heavier
$A_1 > A_2$ then A_1 will be heavier
$A_1 < A_2$ then A_2 will be heavier
Case 3 : $A_1A_2A_3 < B_1B_2B_3$
This time one of the $B_1B_2B_3$ will be heavier, So again as the above case weighting will be done.
2nd weighting $\rightarrow$ B_1 is kept one side and B_2 on the other
if $B_1 = B_2$ B_3 will be heavier
$B_1 > B_2$ B_1 will be heavier
$B_1 < B_2$ B_2 will be heavier
So, as described above, in all the three cases weighting is done only two times to give out the result so minimum no. of weighting required = 2.

MCQ 1.64 The data given in the following table summarizes the monthly budget of an average household.

Category	Amount (Rs.)
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentages of the monthly budget **NOT** spent on savings is
(A) 10% (B) 14%
(C) 81% (D) 86%

SOL 1.64 Option (D) is correct.

$$\text{Total budget} = 4000 + 1200 + 2000 + 1500 + 1800$$
$$= 10,500$$

The amount spent on saving $= 1500$

So, the amount not spent on saving

$$= 10,500 - 1500 = 9000$$

So, percentage of the amount

$$= \frac{9000}{10500} \times 100\%$$
$$= 86\%$$

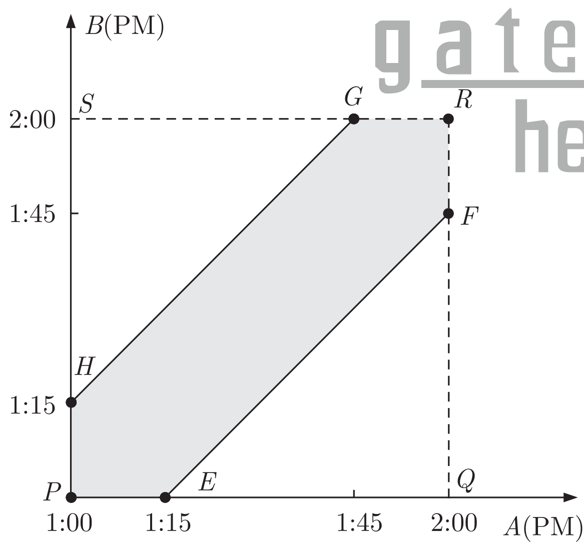
MCQ 1.65 A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a conditions that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that days is

(A) 1/4 (B) 1/16

(C) 7/16 (D) 9/16

SOL 1.65 Option (C) is correct.

The graphical representation of their arriving time so that they met is given as below in the figure by shaded region.



So, the area of shaded region is given by

$$\text{Area of } \square PQRS - (\text{Area of } \Delta\, EFQ + \text{Area of } \Delta\, GSH)$$
$$= 60 \times 60 - 2\left(\frac{1}{2} \times 45 \times 45\right)$$
$$= 1575$$

So, the required probability $= \dfrac{1575}{3600} = \dfrac{7}{16}$